

Distributed filter state estimation of topological random switching in WSN

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Abstract—A distributed filter design method for WSN with sensor nonlinearity saturation and random switching of network topology are proposed in this paper. A mass of sensor nodes are deployed in the sensor network, the target device is sensed and measured, and transmitted to the distributed filter through the network. In the filtering network, the local estimator receives the measurement information from the sensor node. The estimation from the neighboring nodes via a random time-varying topology to complete the state estimation and trajectory tracking for the objective system. The Bernoulli binary distribution is used to describe the random saturation nonlinearity of the sensor network, and the inhomogeneous Markov chain is adopted to represent the random switching topologies. The sufficient conditions are given as distributed filters in the form of linear matrix inequalities method. In the end, the effectiveness of this design method is illustrated by a simulation example.

Key word: Distributed state estimation; H_∞ function; Random switching; Inhomogeneous Markov chain; saturated nonlinearity

I Introduction

A mass of sensor nodes distributed in the detection area constitute a wireless sensor network. It is widely available for environmental monitoring and protection, intelligent equipment, military monitoring and other fields. It has attracted the attention of more and more relevant researchers^{[1]-[5]}. In these many applications, design a distributed estimation algorithm for monitoring target or system has become a basic WSN-based problem. It can be used to estimate and track intended signals, states or tracks. In previous years, many research achievements have been made on distributed filtering algorithms based on WSN^{[6]-[9]}.

At present, most of the existing filter results need to make strict assumptions about the linearity of the sensors. WSN are generally deployed in complex environments, including many uncontrollable factors, such as mountains, highlands and complex oceans, which may lead to random measurement saturation nonlinearity^{[6]-[8]}. Reference [9] solves the problem of output regulation problem of linear heterogeneous multi-agent systems under switched topology. Reference [10] designed a powerful distributed robust filter based on an adaptive event

trigger mechanism. Therefore, the design of distributed robust filtering method with random measurement saturation in wireless sensor networks has practical engineering significance.

It is significance to study a distributed filter state estimation design method with random switching topology under sensor saturation constraints. This paper's main contributions are as follows: Based on sector bounded theory, Bernoulli random variables are used to describe the randomly occurring measurement saturation nonlinearity in WSN, and the designed distributed filter has strong robustness; Based on the non-homogeneous Markov theory, the random change of communication topology is modeled, and the sufficient conditions for the existence of distributed filters with random switching of network topology are obtained, while ensuring the expected performance of the filtering error dynamic system H_∞ .

II Represent of this problem

The data information exchange between sensor nodes are usually represented by a directed graph $G^{r(k)} = (V, E, \partial^{r(k)})$, where is the node set $V = \{1, 2, \dots, N\}$, $E \subseteq V \times V$ is the boundary, and $\partial^{r(k)} = [a_{ij}^{r(k)}]_{N \times N}$ is the adjacency matrix. If the digraph $G^{r(k)}$ has a boundary from sensor node j to sensor node i , and the ordered pair $(i, j) \in E$, $a_{ij}^{r(k)} > 0$, that node is named node i an adjacent node of node j . The definition matrix $L \underline{x}^{(k)} = \left(l_{ij}^{r(k)} \right)_{N \times N}$ is the Laplacian matrix, where $l_{ii}^{r(k)} = \sum_{j \in N_1^{r(k)}} a_{ij}^{r(k)}$, $l_{ij}^{r(k)} = -a_{ij}^{r(k)}$, $\forall i \neq j$. Moreover, assume that for all $i \in V$, $a_{ii}^{r(k)} = 0$. The set formed by all adjacent nodes of node i is named the set of adjacent nodes of node i , denoted $N_1^{r(k)} = \{j \in V: (i, j) \in E\}$

Markov chain $r(k)$ is used to describe the current network topology type. It takes values in a finite set $S = \{1, 2, \dots, n_0\}$, and its time-varying state transition probability matrix is $\Pi(k) = (\pi_{st}(k))_{n_0 \times n_0}$, $\pi_{st}(k)$, It represents the probability that sub topology S jumps to sub topology T , which satisfies

$$\pi_{st}(k) = \text{Prob}(r(k+1) = t | r(k) = s) \quad (1)$$

And $\Pi(k)$ is a time-varying matrix representing an inhomogeneous Markov chain. It's assumed to have multicellular uncertainty, which satisfies $\Pi(k)$

$$\Pi(k) = \sum_{m=1}^{m_0} \alpha_m(k) \Pi^{(m)}, \quad (2)$$

Among them

$$\alpha_m(k) > 0, \sum_{m=1}^{m_0} \alpha_m(k) = 1. \quad (3)$$

Here $\pi_{st} \geq 0$, and for all $s, t \in S$, $\sum_{t=1}^{n_0} \pi_{st} = 1$.

Consider the following discrete-time linear time-invariant system

$$\begin{cases} i(k+1) = Ai(k) + By(k), \\ j(k) = Mi(k), \end{cases} \quad (4)$$

$i(k) \in \mathbb{R}^{n_x}$ is the state vector of the system, $j(k) \in \mathbb{R}^{n_z}$ is the output vector to be estimated, $y(k) \in \mathbb{R}^{n_w}$ is the external interference, and belongs to $l_2[0, \infty)$.

The measurement type of each sensor node in WSN is described as

$$\begin{cases} y_i(k) = C_i x(k), \\ y_{\phi,i}(k) = (1 - \theta_i(k)) \phi_i(y_i(k)) \\ \quad + \theta_i(k) y_i(k) + D_i v_i(k), \end{cases} \quad (5)$$

$y_i(k) \in \mathbb{R}^{n_y}$ is the measurement vector of node i , $v_i(k)$ is the measurement noise, and belongs to $l_2[0, \infty)$. $f(x)$ is the saturation function, which satisfies

$$(\phi_i(y_i(k)) - K_{1,i} y_i(k)) \left(\phi_i(y_i(k)) - K_{2,i} y_i(k) \right) \leq 0 \quad (6)$$

One of them $K_i \triangleq K_{2,i} - K_{1,i}$. In addition, the coefficient matrices A, B, M, C_i, D_i all have corresponding dimensions.

It is assumed $\{\theta_i(k)\}$ that Bernoulli distributed random white sequence with values in $\{0, 1\}$, which is used to describe the randomly occurring saturated nonlinearity and satisfies

$$\begin{aligned} \mathbb{E}\{\theta_i(k)\} &= \text{Prob}\{\theta_i(k) = 1\} = \beta_i, \\ \mathbb{E}\{\theta_i(k) = 0\} &= 1 - \beta_i, \\ \mathbb{E}\{(\theta_i(k) - \beta_i)^2\} &= \beta_i(1 - \beta_i) = \alpha_i, \end{aligned} \quad (9)$$

And assume that for any $1 \leq i \leq N, 0 \leq k < \infty$, the random $\theta_i(k)$ variables are independent of each other.

Construct the following filter subject to random sensor saturation constraints and communication topology switching

$$\begin{cases} \hat{x}_i(k+1) = A\hat{x}_i(k) + L_i^s(y_{\phi,i}(k) - C_i\hat{x}_i(k)) \\ \quad + W_i^s \sum_{j \in \mathcal{N}_i^s} a_{ij}^s (\hat{x}_j(k) - \hat{x}_i(k)), \\ \hat{z}_i(k) = M\hat{x}_i(k), \end{cases} \quad (10)$$

Where $\hat{x}_i \in \mathbb{R}^{n_x}$ is the filter state and $\hat{z}_i(k) \in \mathbb{R}^{n_z}$ is the estimated value of the filter i to $z(k)$. The matrix L_i^s, W_i^s is the filter parameters that need to be determined.

Define the state estimation error as $e_i(k) = x(k) - \hat{x}_i(k)$, then each node state error is described as

$$\begin{aligned} e_i(k+1) &= (A + L_i^s C_i) e_i(k) + Bw(k) \\ &\quad - (1 - \beta_i) L_i^s \phi_{sat,i}(y_i(k)) \\ &\quad + L_i^s (\theta_i(k) - \beta_i) \phi_{sat,i}(y_i(k)) \\ &\quad - L_i^s (K_{1,i} C_i - \beta_i K_{1,i} C_i + \beta_i C_i - C_i) x(k) \\ &\quad - L_i^s (\theta_i(k) - \beta_i) (C_i - K_{1,i} C_i) x(k) \\ &\quad - L_i^s D_i v_i(k) - W_i^s \sum_{j \in \mathcal{N}_i^s} a_{ij}^s (e_i(k) - e_j(k)). \end{aligned} \quad (11)$$

If the output estimation error is defined as $\tilde{z}_i(k) = z(k) - \hat{z}_i(k)$, the output estimation error is described as

$$\tilde{z}_i(k) = M e_i(k). \quad (12)$$

In order to express conveniently, considering N nodes at the same time.

The estimation error vector is

The system state vector is $\bar{x}(k) = [x^T(k), x^T(k), \dots, x^T(k)]^T$

The filter state vector is $\hat{x}(k) = [\hat{x}_1^T(k), \hat{x}_2^T(k), \dots, \hat{x}_N^T(k)]^T$

The output the estimation error vector is

$$\tilde{z}(k) = [\tilde{z}_1^T(k), \tilde{z}_2^T(k), \dots, \tilde{z}_N^T(k)]^T.$$

The sensor network measurement vector is

$$y(k) = [y_1^T(k), y_2^T(k), \dots, y_N^T(k)]^T$$

The measurement noise vector is

$$v(k) = [v_1^T(k), v_2^T(k), \dots, v_N^T(k)]^T$$

The parameter vector of the system is

$$\begin{aligned} \bar{I} &= \text{diag}\{I, I, \dots, I\} \\ \bar{J} &= [J^T, J^T, \dots, J^T]^T \\ \bar{N} &= \text{diag}\{N_1, N_2, \dots, N_N\} \\ \bar{E} &= \text{diag}\{E_1, E_2, \dots, E_N\}, \\ \bar{F} &= \text{diag}\{F, F, \dots, F\} \\ \bar{\beta} &= \text{diag}\{\beta_1, \beta_2, \dots, \beta_N\}, \\ \bar{\alpha} &= \text{diag}\{\alpha_1, \alpha_2, \dots, \alpha_N\} \\ \bar{L}_1 &= \text{diag}\{L_{1,1}, L_{1,2}, \dots, L_{1,N}\} \\ \bar{L}_2 &= \text{diag}\{L_{2,1}, L_{2,2}, \dots, L_{2,N}\} \\ \bar{\theta}(k) &= \text{diag}\{\theta_1(k), \theta_2(k), \dots, \theta_N(k)\} \\ \bar{\phi}_{sat}(m) &= \text{diag}\{\phi_{sat,1}(x_1(m)), \phi_{sat,2}(x_2(m)), \dots, \phi_{sat,N}(x_N(m))\} \end{aligned} \quad (13)$$

The following filtering error system can be obtained

$$\begin{cases} e(k+1) = (\bar{A} - \bar{L}^s \bar{C} - \bar{W}^s \bar{L}^s) e(k) + \bar{B} w(k) - \bar{L}^s \bar{D} v(k) \\ \quad - (I - \bar{\beta}) \bar{L}^s \bar{\phi}_{sat}(k) + (\bar{\theta}(k) - \bar{\beta}) \bar{L}^s \bar{\phi}_{sat}(k) \\ \quad - \bar{L}^s (\bar{K}_1 \bar{C} - \bar{\beta} \bar{K}_1 \bar{C} + \bar{\beta} \bar{C} - \bar{C}) \bar{x}(k) \\ \quad - (\bar{\theta}(k) - \bar{\beta}) \bar{L}^s (\bar{C} - \bar{K}_1 \bar{C}) \bar{x}(k) \\ \tilde{z}(k) = \bar{M} \eta(k), \end{cases} \quad (14)$$

Among them

$$\begin{aligned} \bar{W}^s &= \text{diag}\{W_1^s, W_2^s, \dots, W_N^s\}, \\ \bar{L}^s &= \text{diag}\{L_1^s, L_2^s, \dots, L_N^s\}. \end{aligned} \quad (15)$$

Definition $\eta(k) = [\bar{x}^T(k) \quad e^T(k)]^T$ and $\tilde{\phi}_{sat}(k) = [0 \quad \bar{\phi}_{sat}^T(k)]^T$, according to Equations (4), (5) and (14), the

following augmented filtering error system can be obtained

$$\begin{cases} \eta(k+1) = \mathcal{A}_1^s \eta(k) + (\bar{\theta}(k) - \bar{\beta}) \mathcal{A}_2^s \eta(k) \\ \quad + \mathcal{C}_1^s \tilde{\phi}_{sat}(k) + (\bar{\theta}(k) - \bar{\beta}) \mathcal{C}_2^s \tilde{\phi}_{sat}(k) \\ \quad + \mathcal{B}^s \bar{w}(k), \\ \tilde{z}(k) = \mathcal{M} \eta(k), \end{cases} \quad (16)$$

Among them

$$\begin{aligned} \mathcal{A}_1^s &= \begin{bmatrix} \bar{A} & 0 \\ \Phi_{21}^s & \Phi_{22}^s \end{bmatrix}, \mathcal{A}_2^s = \begin{bmatrix} 0 & 0 \\ \Gamma_{21}^s & 0 \end{bmatrix}, \\ \mathcal{C}_1^s &= \begin{bmatrix} 0 & 0 \\ 0 & -(I - \bar{\beta}) \bar{L}^s \end{bmatrix}, \mathcal{C}_2^s = \begin{bmatrix} 0 & 0 \\ 0 & \bar{L}^s \end{bmatrix}, \\ \mathcal{B}^s &= \begin{bmatrix} \bar{B} & 0 \\ \bar{B} & -\bar{L}^s \bar{D} \end{bmatrix}, \mathcal{M} = [0 \quad \bar{M}]. \end{aligned} \quad (17)$$

here

$$\begin{aligned} \Phi_{21}^s &= -\bar{L}^s (\bar{K}_1 \bar{C} - \bar{\beta} \bar{K}_1 \bar{C} + \bar{\beta} \bar{C} - \bar{C}), \\ \Phi_{22}^s &= \bar{A} - \bar{L}^s \bar{C} - \bar{W}^s \bar{L}^s, \\ \Gamma_{21}^s &= -\bar{L}^s (\bar{C} - \bar{K}_1 \bar{C}). \end{aligned} \quad (18)$$

III Distributed filtering analysis \mathbf{H}_∞

Theorem 1. Given a desired level of perturbation decay $\gamma (\gamma > 0)$, if positive definite matrices $P^s > 0$ exist,

satisfy $\eta^T(0)P^s\eta(0) \leq \gamma^2\eta^T(0)R^s\eta(0)$, $s = 1, 2, \dots, n_0$, and inequalities

$$\begin{bmatrix} -P^s & * & * & * & * & * \\ 0 & -\gamma^2 I & * & * & * & * \\ \tilde{C}_K & 0 & -2I & * & * & * \\ \tilde{P}^s A_1^s & \tilde{P}^s B^s & \tilde{P}^s C_1^s & -\tilde{P}^s & * & * \\ \sqrt{\alpha}\tilde{P}^s A_2^s & 0 & \sqrt{\alpha}\tilde{P}^s C_2^s & 0 & -\tilde{P}^s & * \\ M & 0 & 0 & 0 & 0 & -I \end{bmatrix} < 0, \quad (22)$$

Where $\tilde{P}^s = \sum_{t=1}^{n_0} \tilde{\pi}_{st} P^t$, $\tilde{\pi}_{st} = \max_{1 \leq m \leq m_0} \{\pi_{st}^m\}$, $\tilde{C}_K = \begin{bmatrix} 0 & 0 \\ (\tilde{K}_2 - \tilde{K}_1)\tilde{C} & 0 \end{bmatrix}$, the augmented filter error system (16) has a given H_∞ performance.

Proof, definition $\tilde{K} = \tilde{K}_2 - \tilde{K}_1$, and performance analysis function

$$J = \eta^T(k+1)P^t\eta(k+1) - \eta^T(k)P^s\eta(k). \quad (23)$$

According to the saturation constraint condition (5), it can be obtained

$$-2\tilde{\phi}_{sat}^T(k)\tilde{\phi}_{sat}(k) + 2\tilde{\phi}_{sat}^T(k)\tilde{K}y(k) > 0. \quad (24)$$

Then get

$$\xi(k) = -2\tilde{\phi}_{sat}^T(k)\tilde{\phi}_{sat}(k) + 2\tilde{\phi}_{sat}^T(k)\tilde{C}_K\eta(k) > 0. \quad (25)$$

Introduce the constant zero equality

$$\begin{aligned} \tilde{z}^T(k)\tilde{z}(k) - \gamma^2\tilde{w}^T(k)\tilde{w}(k) - \tilde{z}^T(k)\tilde{z}(k) + \\ \gamma^2\tilde{w}^T(k)\tilde{w}(k) = 0. \end{aligned} \quad (26)$$

Define the combination (23) and (25), then have

$$\begin{aligned} E\{J(k)\} \leq E\{\xi^T(k)\Lambda^s\xi(k) - \tilde{z}^T(k)\tilde{z}(k) \\ + \gamma^2\tilde{w}^T(k)\tilde{w}(k)\}, \end{aligned} \quad (27)$$

Among them

$$\begin{aligned} \Lambda^s &= \begin{bmatrix} \Lambda_{11}^s & * & * \\ \Lambda_{21}^s & \Lambda_{22}^s & * \\ \Lambda_{31}^s & \Lambda_{32}^s & \Lambda_{33}^s \end{bmatrix}, \\ \Lambda_{11}^s &= A_1^{sT}\tilde{P}^s A_1^s + \alpha A_2^{sT}\tilde{P}^s A_2^s - P^s + M^T M, \\ \Lambda_{21}^s &= B^{sT}\tilde{P}^s A_1^s, \Lambda_{22}^s = B^{sT}\tilde{P}^s B^s - \gamma^2 I, \\ \Lambda_{31}^s &= C_1^{sT}\tilde{P}^s A_1^s + \alpha C_2^{sT}\tilde{P}^s A_2^s + \tilde{C}_K, \\ \Lambda_{32}^s &= C_1^{sT}\tilde{P}^s B^s, \Lambda_{33}^s = C_1^{sT}\tilde{P}^s C_1^s + \alpha C_2^{sT}\tilde{P}^s C_2^s - 2I. \end{aligned} \quad (28)$$

Then, add both sides of inequality (27) from 0 to N-1, and we can get

$$\begin{aligned} \sum_{k=0}^{N-1} E\{J(k)\} \leq E\{\eta^T(N)\tilde{P}^s\eta(N) - \eta^T(0)P^s\eta(0) \\ \leq E\left\{\sum_{k=0}^{N-1} \xi^T(k)\Lambda^s\xi(k)\right\} \\ - E\left\{\sum_{k=0}^{N-1} (\tilde{z}^T(k)\tilde{z}(k) - \gamma^2\tilde{w}^T(k)\tilde{w}(k))\right\}. \end{aligned} \quad (29)$$

Then the performance constraints H_∞ defined in (19) can be further described as

$$\begin{aligned} J \leq E\left\{\sum_{k=0}^{N-1} \xi^T(k)\Lambda^s\xi(k)\right\} - E\{\eta^T(N)\tilde{P}^s\eta(N)\} \\ + \eta^T(0)(P^s - \gamma^2 R^s)\eta(0). \end{aligned} \quad (30)$$

According to Schur's complement lemma, inequality (22) contains $\Lambda^s < 0$, while $\tilde{P}^s > 0$, and the initial conditions $P^s \leq \gamma^2 R^s$, then we have $J < 0$, and the proof ends.

4 Simulation examples

Through a numerical simulation example, we verify the effectiveness of the proposed method.

Suppose a sensor network consisting of 4 nodes, and its communication topology is shown in Figure 1.

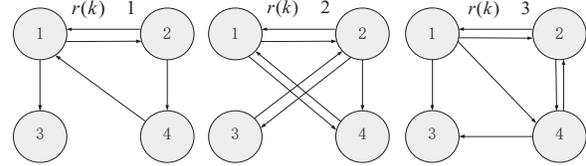


Figure 1 Topology of sensor network

Assuming that the topology switch has nonhomogeneous Markov randomness, its transition probability matrix is composed of the following three matrices $\Pi(k)$

$$\begin{aligned} \Pi^{(1)} &= \begin{bmatrix} 0.5 & 0.3 & 0.2 \\ 0.3 & 0.4 & 0.3 \\ 0.2 & 0.3 & 0.5 \end{bmatrix}, \Pi^{(2)} = \begin{bmatrix} 0.3 & 0.3 & 0.4 \\ 0.5 & 0.3 & 0.2 \\ 0.3 & 0.3 & 0.4 \end{bmatrix}, \\ \Pi^{(3)} &= \begin{bmatrix} 0.5 & 0.2 & 0.3 \\ 0.3 & 0.3 & 0.4 \\ 0.3 & 0.4 & 0.3 \end{bmatrix}. \end{aligned}$$

The system parameters are chosen as

$$A = \begin{bmatrix} 0.3 & 0.1 \\ -1 & 0.2 \end{bmatrix}, B = \begin{bmatrix} 0.3 \\ 0.1 \end{bmatrix}, M = \begin{bmatrix} 2 & 2 \end{bmatrix}.$$

Sensor network measurement parameters are

$$\begin{aligned} C_1 = [5 \ 4], C_2 = [3 \ 3], C_3 = [4 \ 5], C_4 = [3 \ 4], \\ D_1 = 0.3, D_2 = 0.1, D_3 = 0.2, D_4 = 0.1. \end{aligned}$$

The saturated nonlinearity is described as

$$\begin{aligned} \phi_{sat,i}(y_i(k)) &= 0.5(K_{1,i} + K_{2,i})y_i(k) \\ &+ 0.5(K_{2,i} - K_{1,i})\sin(y_i(k)) \end{aligned}$$

Among them

$$\begin{aligned} K_{1,1} = K_{1,3} = 0.6, K_{1,2} = K_{1,4} = 0.7, \\ K_{2,1} = K_{2,2} = 0.8, K_{2,3} = K_{2,4} = 0.9. \end{aligned}$$

In addition, the probability of random $\beta_i = 0.8$, $i = 1, 2, 3, 4$ nonlinearity in this sensor network is assumed, the system interference input is $w(k) = e^{-0.2k} \sin(k)$, and the measurement noise is $v_i(k) = \frac{1}{k^s}$, $i = 1, 2, 3, 4$. It is assumed that the initial state of the system and the initial state of the filter are 0, and the topology of the initial filter network is $r(0) = 1$.

The optimization problem is optimized by using MATLAB yalmip toolbox, and the optimal solution $\gamma = 5.0485T$ is obtained. The simulation results are shown in Figure 2,3.

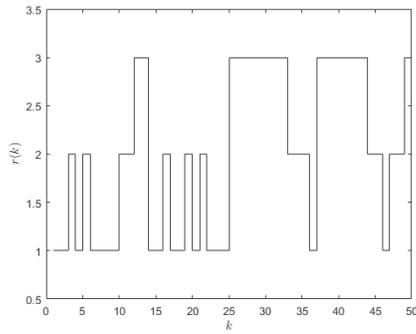


Figure 2 Markov chain $\{r(k)\}$

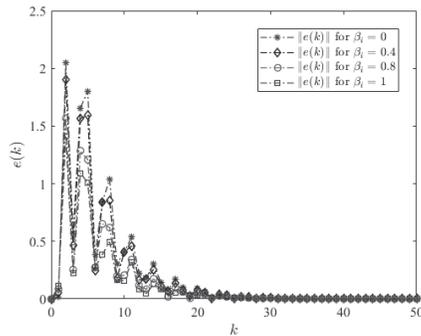


Figure 3 Filter network estimation error $e_i(k), i = 1,2,3,4$.

Figure 2 shows the evolution process of a Markov chain $r(k)$. Figure 3 depicts the comparison curve of filter network error when β_i is taken as 0, 0.4, 0.8 and 1 respectively.

Table 1 Different relationships with system robust performance, $\beta_i, i = 1,2,3,4$

b_i	0	0.2	0.4	0.6	0.8	1
g	5.98	5.84	5.65	5.41	5.04	4.51

The table 1 reveals the influence of selecting distinct β_i on the robust performance of system H_∞ . According to figure 3 and table 1, it can be analyzed that the saturation nonlinearity of the sensor network affects the robustness of the system. When the WSN does not have the measurement saturation nonlinearity, the robustness of the system is the prime, and when the WSN completely has the measurement saturation nonlinearity, the robustness of this system is the worst. Contrasted with these two extreme situations, when the sensor network has the measurement saturation nonlinearity with a certain probability, The system attains better filtering accuracy while ensuring strong robustness.

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