



showed a downward trend. The overall mortality rate tends to be flat, showing a slight upward trend after 2006, and the aging of population may be the reason for the increase in mortality. As shown in Figure 3, many factors affect the population structure of the system, such as environment, economy and so on. According to the characteristics of China's population development over the years, it is found that its growth trend is similar to the Logistic curve, so the Logistic Model can be used to simulate the long-term trend of China's population development.

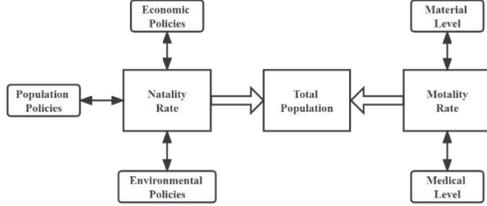


Figure 3. The Factors which affecting Population Structure of China.

Assuming that the population of the year  $t_0$  is  $P(0)$ , the growth rate is  $r$ , then the population of the year  $t$  is calculated as follows:

$$P(t) = P(0)(1 + r)^t \quad (1)$$

The exponential form is as follows:

$$P(t) = P_0 e^{rt} \quad (2)$$

Then we take the derivative and get the homogeneous equation:

$$\frac{dP(t)}{dt} = rP(t) \quad (3)$$

Considering that natural resources, environmental conditions and other factors block population growth, and the blocking effect increases with the increase of population, that is, the population growth rate  $r$  is the decreasing function of the total population  $P(t)$ . Therefore, by modifying the above equation 2.3 and adding a quadratic term  $P(t)^2$  which characterizes the environmental constraint factor, the second order Bernoulli homogeneous equation is obtained as follows:

$$\frac{dP(t)}{dt} = rP(t) - qP(t)^2 = rP(t)\left[1 - \frac{P(t)}{P_m}\right] \quad (4)$$

Here  $q$  is the constraint parameter and  $P_m = \frac{r}{q}$  represents the saturated population of the region, that is, the maximum population capacity. The initial condition and saturation condition of the equation are  $P(t_0) = P_0$  and  $P(t) \leq P_m$ . So the Logistic prediction model is obtained as follows:

$$P(t) = \frac{P_m}{1 + \left(\frac{P_m}{P_0} - 1\right)e^{-rt}} = \frac{P_m}{1 + \lambda e^{-rt}} \quad (5)$$

### III. GM(1,1) GREY PREDICTION MODEL

Grey prediction is a kind of uncertain system prediction theory, and the prediction model based on GM(1,1) has been widely concerned and studied<sup>[5]</sup>. Accumulative Generation Operation is the most important feature of grey system theory, and its purpose is to reduce the randomness of data. Grey prediction only needs the least data to construct the grey differential equation for prediction, and its main advantage is using as few as four

data items to predict future data.[5] Grey prediction has been widely used in engineering science, social science, agriculture, fertility, power consumption, management and other fields.

Suppose that the time series  $X^{(0)}$  has  $n$  observations:  $X^{(0)} = \{X^{(0)}(1), X^{(0)}(2), \dots, X^{(0)}(n)\}$ , and the new series is generated by accumulation:  $X^{(1)} = \{X^{(1)}(1), X^{(1)}(2), \dots, X^{(1)}(n)\}$ . Then the corresponding differential equation of the GM(1,1) model is as follows:

$$\frac{dX^{(1)}}{dt} + \alpha X^{(1)} = \mu \quad (6)$$

Here  $\alpha$  is developmental grey number, and  $\mu$  is endogenous control grey number. Set  $\hat{\alpha}$  as the parameter vector to be estimated,  $\hat{\alpha} = \begin{bmatrix} \alpha \\ \mu \end{bmatrix}$ , which can be solved by the least square method. The solution is as follows:

$$\hat{\alpha} = (B^T B)^{-1} B^T Y_n \quad (7)$$

By solving the differential equation, the prediction model can be obtained as follows:

$$\hat{X}^{(1)}(k+1) = \left[X^{(0)}(1) - \frac{\mu}{\alpha}\right] e^{-\alpha k} + \frac{\mu}{\alpha}, k = 0, 1, 2, \dots, n \quad (8)$$

### IV. ANALYTIC HIERARCHY PROCESS MODEL

The Analytic Hierarchy Process (AHP) was first developed by Thomas L. Saaty in the 1970s. The algorithm is a combination of qualitative and quantitative decision analysis method, which helps decision makers to make some complex problems organized and hierarchical, and quantify some qualitative problems from the perspective of psychology. By using this method, the comparison and calculation of various factors can be realized, and finally the weights of different schemes can be obtained, which provides a basis for the selection of the best scheme<sup>[6]</sup>.

AHP can make the complex problem hierarchical, and decompose the problem into different levels according to the nature of the problem and the overall objective, and forming a multi-level analytical structure model, which is generally divided into target layer, decision-making layer and scheme layer. The target layer of the elderly service beds problem is the Best model, and the decision-making layer is the government and business. The scheme layer is composed of government subsidy, social employment, social contributions, operating income and financing income.

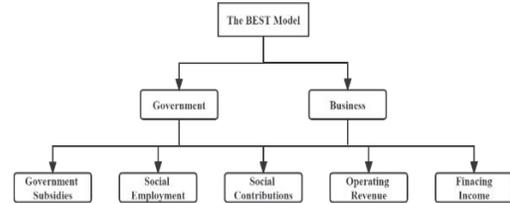


Figure 4. AHP of the best model for elderly service beds.

Step1: Using AHP to analyze the problem requires the construction of a judgment matrix first. Assumed that we want to compare the impact of  $n$  factors  $X = \{x_1, \dots, x_n\}$  on factor  $Z$ , we need to use the psychological judgment model to compare each factor in pairs. That is, each time we take two factors  $x_i$  and  $x_j$ , use  $a_{ij}$  to express the ratio of the size of the impact of  $x_i$  and  $x_j$  on factor  $Z$ , then we can construct the matrix  $A$  as the judgment matrix between factor  $Z$  and  $X$ :

$$A = \begin{pmatrix} a_{11} & \dots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{n1} & \dots & a_{nn} \end{pmatrix} \quad (9)$$

Step2: The judgment matrix  $A$  corresponds to the eigenvector  $W$  of the eigenvalue  $\lambda$ , and whether the eigenvector  $W$  can be used as a weight vector needs to be checked. If it can meet the consistency requirements, the eigenvector corresponding to the non-zero eigenvalue can be normalized as the weight vector. On the contrary, within a certain allowable range, the normalized eigenvector corresponding to the maximum eigenvalue can be used as the weight vector. For the determination of the allowable range, it is necessary to define an index  $CI$  that can evaluate the inconsistent size of the judgment matrix, which can be calculated as  $CI = \frac{\lambda - n}{n - 1}$ . In order to measure the size of  $CI$ , the random consistency index  $RI$  is introduced. Psychologists give the random consistency index ( $RI$ ) as follows:

TABLE I. RANDOM CONSISTENCY INDEX ( $RI$ )

$n$	1	2	3	4	5	6	7	8	9	10	11
$RI$	0	0	0.58	0.9	1.12	1.24	1.32	1.41	1.45	1.49	1.51

Step3: Use  $CI$  and  $RI$  to get  $CR = \frac{CI}{RI}$ , the consistency test is satisfied when  $CR \leq 0.1$ . On the contrary, the judgment matrix need to be reconstructed, and then the hierarchical single ranking and consistency test are carried out until consistency test is satisfied.

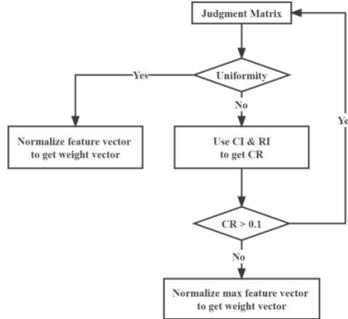


Figure 5. Flow Chat of AHP.

## V. MODEL SOLUTION

### A. Solution of Logistic Model

In order to predict the number of people with elderly service needs, combined with the population number and structure given in the attachment, we show the number of elderly people in each year, and the data were sorted out and put into the regression model.

The 2011-2021 China population over 65 Years Old trend chart is shown in Figure 6.

From Figure 6 we can see the number of population aged over 65 in China is increasing year by year.

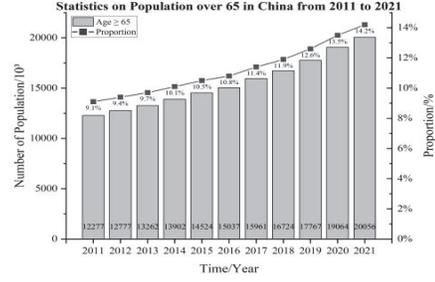


Figure 6. The Number and Proportion of the Population aged over 65 in China from 2011 to 2021.

Referring to the attachment and the actual number of middle-aged and elderly people in relevant data, the formula for population growth can be calculated with  $P(t) = \frac{P_m}{1 + \lambda e^{-rt}}$ . The parameters  $P_m, \lambda, r$  can be calculated by the population data in the previous section. After calculation, the formula can be obtained as Formula (10), and the Logistic fitting is shown in Figure 8.

$$P(t) = \frac{6.3001e+07}{1 + (7.5937e+07) \times e^{-0.0419t}} \quad (10)$$

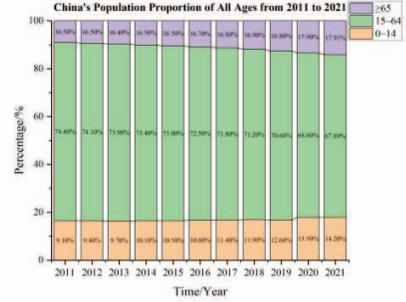


Figure 7. Population structure of Different age groups in China from 2011 to 2021.

In Figure 8, with year as the horizontal axis and population as the vertical axis, the scatter graph (blue points) and its fitting curve (black curve) are obtained. It can be seen that the fitting graph from 2006 to 2019 has a good effect.

Through the above fitting curve, it is predicted that in 2022 and 2023, the number of elderly people over 65 will be 200.15 million and 208.72 million. Therefore, according to the survey, 10% of the elderly people are willing to join, so the total market size of elderly service will be about 200 million.

According to a report on the pension service industry of 2020, about 90 percent of the elderly intend to provide care at home, while 6 percent choose to provide care in the community and 4 percent choose to provide care in institutions. Therefore, the market capacity of the above three pension modes available is currently 18 million, 1.2 million and 0.8 million beds.

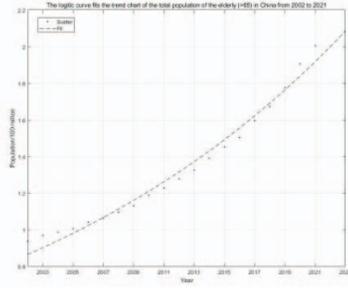


Figure 8. Logistic Fit Curve.

### B. Solution of GM(1,1) Grey Prediction Model

According to the report of pension institutions compiled by National Bureau of Statistics and China Business Industry Research Institute, there were 30.9 beds for every 1,000 elderly people in 2018<sup>[7]</sup>, which is an increase of 2% compared with 2017.

The number of beds in elderly nursing homes per 1,000 users is taken as the forecast target, and we plug in the data to the grey prediction model to get the prediction results.

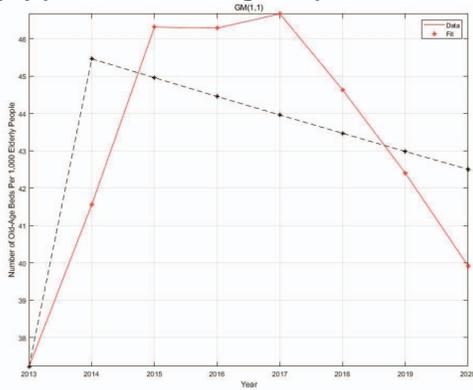


Figure 9. Prediction trend of elderly service beds per 1000 users.

The prediction we get shows that by 2019, there will be 35.1 beds for every 1,000 users. Compared with 4% of the elderly people who are willing to live in commercial institutions, that is, the number of beds should be at least 40 for every 1000 users under the above condition. Therefore, institutional pension market is far from saturated, there is a large market opportunity for elderly service.

### C. Prediction Error Analysis of GM(1,1) Model

According to the prediction model to get  $\hat{X}^{(1)}(i)$ , and it's subtracted cumulatively to generate the sequence  $\hat{X}^{(0)}(i)$ . Finally, the absolute and relative error sequences of the original sequence  $X^{(0)}(i)$  and sequence  $\hat{X}^{(0)}(i)$  are given. In this paper, the prediction errors of the model are tested by means of variance ratios.

Standard deviation of the original series is as follows:

$$S_1 = \sqrt{\frac{\sum [X^{(0)}(i) - \bar{X}^{(0)}]^2}{n-1}} \quad (11)$$

Standard deviation of the absolute error series is as follows:

$$S_2 = \sqrt{\frac{\sum [d^{(0)}(i) - \bar{d}^{(0)}]^2}{n-1}} \quad (12)$$

The variance ratio  $C$  is obtained by  $C = \frac{S_2}{S_1}$ . The calculated  $C$  value is:  $C = 0.659 \in (0.5, 0.85)$ , which satisfies the variance ratio test.

### D. Solution of AHP

By the end of 2019, China had a total of about 200,000 institutional facilities for various types of elderly services, an increase of 15.4% year-on-year, 32,000 community elderly structures and facilities, over 34,000 elderly institutions, 7.61 million elderly service beds, an increase of 1.2% over previous years, 3.50 elderly service beds per 100 elderly people, and the proportion of elderly service beds to the elderly population was 3.50%<sup>[8]</sup>. Compared with other countries, the proportion of elderly service beds to the elderly population is 28% in the United States and 13% in Europe. In other developed countries, the proportion of elderly people in elderly institutions to all elderly people ranges from about 9% to 11%, which shows that there is a large gap in the number of elderly service beds that can be provided in China.

Through research, this paper collects data on the number of service institutions (10,000), number of beds (10,000), number of elderly service beds per 1,000 elderly people, tertiary industry (100 million yuan), number of elderly people (>65) (10,000), proportion of elderly people (>65), proportion of tertiary industry, number of elderly people served per service institution and number of beds provided per service institution in the annual social service development statistical bulletin from 2013 to 2020<sup>[9]</sup>, and calculates the correlation to give the correlation matrix between indicators, which can visualize the degree of correlation of different indicator data, as shown in Figure 10.

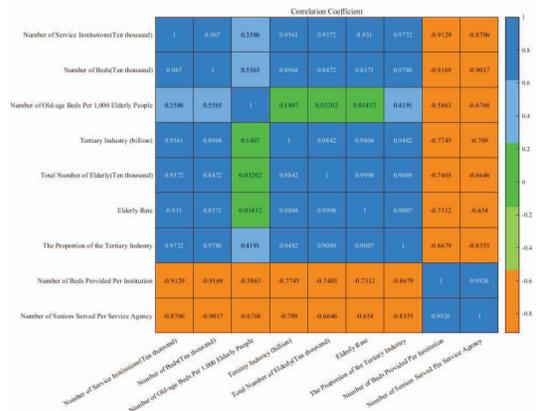


Figure 10. Indicator Correlation Matrix.

In the following, in order to explore the operation model from the government's perspective, this paper will explore the factors that may affect the future operation of elderly service beds through the hierarchical analysis method, and conduct an analysis to obtain government subsidies, social employment, social contributions, operating income and financing income as the main influencing factors. Finally illustrates the proportion of the factors in the best model.

The judgment and weight matrix are constructed, and the eigenvectors with consistency metrics CI and CR are solved as follows:

$$A - B = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}, W_A = \begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix} \quad (13)$$

$$\lambda_{1\max} = 2, CI_1 = 0, CR_1 = 0$$

$$B_1 - C = \begin{bmatrix} 1 & 1 & 1 & 3 & 2 \\ 1 & 1 & 1 & 3 & 2 \\ 1 & 1 & 1 & 3 & 2 \\ 1/3 & 1/3 & 1/3 & 1 & 1 \\ 1/2 & 1/2 & 1/2 & 1 & 1 \end{bmatrix}$$

$$W_{B1} = [0.2612 \quad 0.2612 \quad 0.2612 \quad 0.0951 \quad 0.1211]^T \quad (14)$$

$$\lambda_{2\max} = 5.01980, CI_2 = 0.00495, CR_2 = 0.00440$$

$$B_2 - C = \begin{bmatrix} 1 & 1 & 1 & 1/3 & 1/5 \\ 1 & 1 & 1 & 1/3 & 1/5 \\ 1 & 1 & 1 & 1/3 & 1/5 \\ 3 & 3 & 3 & 1 & 1/2 \\ 1/2 & 1/2 & 1/2 & 2 & 1 \end{bmatrix}$$

$$W_{B2} = [0.0902 \quad 0.0902 \quad 0.0902 \quad 0.2612 \quad 0.4683]^T \quad (15)$$

$$\lambda_{3\max} = 5.00400, CI_3 = 0.00100, CR_3 = 0.00089$$

The combined weight vector of the scheme layer to the target is:

$$W = (0.294, 0.284, 0.146, 0.140, 0.136)^T \quad (16)$$

After calculation, all can pass the consistency test.

In conclusion, government subsidies, social employment, social contributions, operating income and financing income, their share in the best model is 0.294, 0.284, 0.146, 0.14 and 0.136 respectively.

## VI. CONCLUSION

Combined with the parameters and prediction results of the model, the specific suggestions to the elderly service departments are as follows:

The first is to transform some of the nursing homes, subsidize and support them and introduce social forces

through public bidding, self-financing, self-development and so on.

The second is to focus on service-oriented construction. The government can raise the status of elderly service by creating social, professional pension services and care services, so as to attract more elderly people to move in.

The third is to improve the social security system. The elderly service system is the most important aspect of social security. The data show that the support ratio in China is gradually decreasing, from 3.3 to 2.9, and the situation is becoming more and more serious with the aging population. Therefore, the government needs to constantly improve the basic old-age insurance system for urban workers<sup>[10]</sup>, increase the subsidy of the fund, comprehensively build transportation and public service facilities, and support relevant pension enterprises.

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