

Trajectory tracking of unmanned vehicle based on improved model predictive control

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Abstract: Driverless technology has become a hot spot of current research, in order to improve the speed of unmanned vehicle path planning. Firstly, the artificial potential field (APF) model is added to the objective function of model predictive control (MPC) as a path planning term to make path planning and trajectory tracking simultaneously, so as to optimize the model controller. Secondly, by introducing the adjustment factor of objective function to solve the problem that MPC objective function may have no optimal solution, and then linearize the objective function to accelerate the speed of solving the objective function. Finally, Matlab / Simulink and CarSim are used for joint simulation experiments to compare the path planning and tracking effects before and after the improvement and optimization of the controller. The experimental results show that the improved model predictive controller has faster solution speed.

Keywords: artificial potential field; model predictive control; unmanned vehicle; path planning; trajectory tracking;

I. INTRODUCTION

Most of the existing studies are based on APF for local path planning, and then MPC for trajectory tracking, but the real-time performance is not high^[1]. In view of this disadvantage, local path planning and trajectory tracking control are integrated to realize vehicle trajectory tracking at the same time. When designing the objective function of MPC, the APF environment model is added to the objective function as a path planning item, and the adjustment factor is added as an optimal solution guarantee item. Linearize the designed objective function to speed up its solution speed. Finally, through simulation experiments, the effects of path planning and trajectory tracking before and after controller improvement and optimization are compared.

II. TRAJECTORY TRACKING BASED ON MPC

Model predictive control (MPC) is an optimized control method, which has three main characteristics: predictive model, rolling optimization and feedback correction^[2].

A. Vehicle dynamics model

Since the driving scene in this paper is a structured road and the vehicle speed is fast, vehicle dynamics is selected as the predictive model of MPC. The vehicle has three directions of motion, i.e. longitudinal, lateral and yaw

motion^[3]. Fig.1 is a vehicle monorail model used to describe vehicle dynamics.

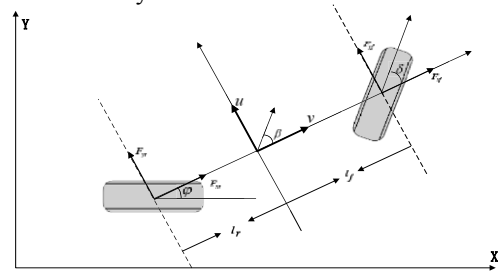


Figure 1. Vehicle monorail model.

According to the force condition of the vehicle, the vehicle dynamics model can be obtained by (1):

$$\begin{cases} ma_x = F_{xf} + F_{xr} \\ mv(\dot{\beta} + \dot{\phi}) = F_{yf} + F_{yr} \\ I_z \ddot{\phi} = l_f F_{yf} - l_r F_{yr} \\ \dot{X} = \dot{x} \sin \phi - \dot{y} \sin \phi \\ \dot{Y} = \dot{x} \cos \phi - \dot{y} \cos \phi \end{cases} \quad (1)$$

When the current wheel angle and tire longitudinal slip ratio are small, the tire force can be approximately described by linear function, so the longitudinal force and lateral force of tire can be obtained by (2):

$$\begin{cases} F_{xf} = C_{xf} s_{fz} \\ F_{xr} = C_{xr} s_{rz} \\ F_{yf} = C_{yf} \alpha_f \\ F_{yr} = C_{yr} \alpha_r \end{cases} \quad (2)$$

α_f, α_r , represent the side deflection angle of front and rear wheels respectively, as shown in (3):

$$\begin{cases} \alpha_f = \delta - \frac{u}{v} - \frac{l_f \dot{\phi}}{v} \\ \alpha_r = \frac{u - l_r \dot{\phi}}{v} \end{cases} \quad (3)$$

F_{xf}, F_{xr} are the forces in the z-axis direction on the front and rear tires of the vehicle, then the forces on the tire in the Y direction can be expressed by (4):

$$\begin{cases} F_{yf} = C_{yf} \alpha_f = C_{yf} \left(\delta - \frac{u}{v} - \frac{l_f \dot{\phi}}{v} \right) \\ F_{yr} = C_{yr} \alpha_r = C_{yr} \left(\frac{u - l_r \dot{\phi}}{v} \right) \end{cases} \quad (4)$$

For the sum of forces in the z-axis direction on the front and rear tires of the vehicle, if the driving speed of the vehicle changes slowly, it can be calculated by (5):

$$\begin{cases} F_{zf} = \frac{l_r mg}{2(l_f + l_r)} \\ F_{zr} = \frac{l_f mg}{2(l_f + l_r)} \end{cases} \quad (5)$$

Based on the above discussion, the following nonlinear vehicle dynamics model can be obtained by (6):

$$\begin{cases} m a_x = C_{yf} s_f + C_{yr} s_r \\ m v (\dot{\beta} + \dot{\phi}) = C_{yf} \left(\delta - \frac{u}{v} - \frac{l_f \dot{\phi}}{v} \right) + C_{yr} \left(\frac{u - l_r \dot{\phi}}{v} \right) \\ I_z \dot{\omega} = l_f C_{yf} \left(\delta - \frac{u}{v} - \frac{l_f \dot{\phi}}{v} \right) - l_r C_{yr} \left(\frac{u - l_r \dot{\phi}}{v} \right) \\ \dot{X} = \dot{x} \sin \varphi - \dot{y} \sin \varphi \\ \dot{Y} = \dot{x} \cos \varphi - \dot{y} \cos \varphi \end{cases} \quad (6)$$

In the established vehicle dynamics model,

$\chi = [a_x, \delta]^T$, is used to represent the selected longitudinal acceleration and the vehicle front wheel Angle position.

(X, Y) , φ , $\dot{\phi}$, u , and v respectively represent geodetic abscissa and ordinate, heading angle, heading angular velocity, lateral velocity and longitudinal velocity. Finally expressed as, $\xi = [X, Y, \varphi, \dot{\phi}, u, v]$.

B. Linearization of vehicle dynamics model

Due to the high requirement of real-time performance of the controller when the unmanned vehicle runs at high speed, linear time-varying model predictive control is adopted.

Firstly, the established vehicle dynamics model can be expressed by (7):

$$\bar{\xi} = g(\xi, \chi) \quad (7)$$

When Taylor expansion is carried out at any point, (ξ_r, u_r) , and only the first-order term is retained, the following can be obtained by (8):

$$\bar{\xi} = g(\xi_r, \chi_r) + \frac{\partial g}{\partial \xi} \bigg|_{\substack{\xi=\xi_r \\ \chi=\chi_r}} (\xi - \xi_r) + \frac{\partial g}{\partial \chi} \bigg|_{\substack{\xi=\xi_r \\ \chi=\chi_r}} (\chi - \chi_r) \quad (8)$$

Equation (8) can be written as (9) by introduced Jacobian determinants $J_g(\xi)$ and $J_g(\chi)$ of function g :

$$\bar{\xi} = g(\xi_r, \chi_r) + J_g(\xi)(\xi - \xi_r) + J_g(\chi)(\chi - \chi_r) \quad (9)$$

At point, (ξ_r, χ_r) , subtract equations (9) and (7) to obtain (10):

$$\bar{\xi} - \xi_r = J_g(\xi)(\xi - \xi_r) + J_g(\chi)(\chi - \chi_r) \quad (10)$$

let $\hat{\xi} = \bar{\xi} - \xi_r$, $A(t) = J_g(\xi)$, $B(t) = J_g(\chi)$, $\hat{\xi} = \xi - \xi_r$,

$\hat{\chi} = \chi - \chi_r$ then equation (10) can be written as (11):

$$\dot{\hat{\xi}} = A(t)\hat{\xi} + B(t)\hat{\chi} \quad (11)$$

Model predictive control is to predict the output in the next period of time based on the current time combined with the measured value, and repeat the above steps at the next time^[4]. The whole process is discrete step by step, and the prediction model (vehicle dynamics model) is continuous, so it is also necessary to discretize the vehicle dynamics model. Discretize equation (11) to obtain (12) and (13):

$$\xi(t+1) = A(t)\xi(t) + B(t)\chi(t) \quad (12)$$

$$\chi(t) = \chi(t-1) + \Delta\chi(t) \quad (13)$$

C. Objective function design

In order to consider factors such as vehicle stability and economic comfort, vehicle speed and control increment are added to the objective function, and in order to ensure that the objective function has an optimal solution, the optimal solution guarantee term is added. The objective function of single vehicle driving is designed as (14):

$$J(\xi(t), \chi(t-1), \Delta\chi(t)) = \sum_{i=1}^{N_p} \|U_{all}(t+i|t)\|_Q^2 + \sum_{i=1}^{N_p} \|V(t+i|t) - V_{des}\|_R^2 + \sum_{i=1}^{N_{c-1}} \|\Delta\chi(t+i-1|t)\|_S^2 + \rho e^2 \quad (14)$$

The first part is the vehicle environmental potential field. By combining the discrete vehicle dynamics model and the control increment input sequence at time t , the potential field value of the vehicle at time $t+i$ in the prediction time domain is calculated^[5]. Through the potential field value, the vehicle risk is evaluated, and the point with small potential field value is selected to be added to the trajectory as the optimal trajectory point for prediction.

D. Objective function optimization

If the designed nonlinear objective function is used directly, the optimization speed of model predictive controller will be slow^[6]. Therefore, the objective function of the model predictive controller is linearized to improve its optimization speed.

At time $t=k$, Taylor expansion is performed as (15):

$$J(\xi(t), \chi(t-l), \Delta\psi(t)) = J(\xi(k), \chi(k-l), \Delta\psi(k)) + \left. \frac{\partial J}{\partial t} \right|_{t=k} (t-k) \quad (15)$$

let $a = J(\xi(k), \chi(k-l), \Delta\psi(k))$, and let $B = \left. \frac{\partial J}{\partial t} \right|_{t=k}$,

$E(\xi(t), \chi(t-l), \Delta\psi(t))$ is introduced as the linearized objective function, the Taylor expansion of the objective function can be written as (16):

$$E(\xi(t), \chi(t-l), \Delta\psi(t)) = b \cdot t + (a - b \cdot k) \quad (16)$$

E. Controller constraint formulation

Since the objective function uses the control increment as the input, linear constraints are added to the control quantity itself and the control increment, that is, linear constraints are imposed on the longitudinal acceleration, a_x , longitudinal acceleration change rate, \dot{a}_x , front wheel steering angle, δ , and front wheel steering angle change rate, $\dot{\delta}$ [7].

The values of the upper and lower bounds of the above related control quantities and increments can be shown in Tab. I.

TABLE I. UPPER AND LOWER BOUND VALUES OF CONTROL QUANTITY AND INCREMENTAL CONSTRAINT PARAMETERS

parameter	minimum	Maximum
Longitudinal acceleration, a_x (M / S ²)	-4	2
Longitudinal acceleration change rate, \dot{a}_x (M / S ³)	-25	25
Front wheel steering angle, δ (Rad)	-15	15
Front wheel angle change rate, $\dot{\delta}$ (Rad/S)	-9.4	9.4

In order to improve the stability of the vehicle and the universality of the controller, the constraints such as centroid deflection angle, vehicle adhesion coefficient, front and rear tire side deflection angle and vehicle speed shall also be considered. Assuming that the driving road is in good condition, the values of, $\beta, K, \alpha_f, \alpha_r, V$ are shown in Tab. II.

TABLE II. VALUES OF UPPER AND LOWER BOUNDS OF, $\beta, K, \alpha_f, \alpha_r, V$

parameter	minimum	Maximum
Centroid deflection angle, β (Rad)	-12	12
K	0.7	1
Sideslip angle of front and rear tires, α_f, α_r (Rad)	-2.5	2.5
Longitudinal speed, V (M / s)	0	30

Combined with the vehicle dynamics model, objective function and constraints, the trajectory planning and tracking of unmanned vehicle based on model predictive control can be described as the optimization problem shown in (17):

$$\begin{aligned} & \min_{\Delta\psi(t)} \{E(\xi(t), \chi(t-1), \Delta\psi)\} \\ & s.t. \quad \chi_{\min}(t) \leq \chi(t) \leq \chi_{\max}(t) \\ & \quad \Delta\chi_{\min}(t) \leq \Delta\chi(t) \leq \Delta\chi_{\max}(t) \\ & \quad \xi(t+1) = A(t)\xi(t) + B(t)\chi(t) \\ & \quad \chi(t) = \chi(t-1) + \Delta\chi(t) \\ & \quad a_{x,\min} \leq a_x \leq a_{x,\max} \\ & \quad \delta_{\min} \leq \delta \leq \delta_{\max} \\ & \quad \dot{a}_{x,\min} \leq \dot{a}_x \leq \dot{a}_{x,\max} \\ & \quad \dot{\delta}_{\min} \leq \dot{\delta} \leq \dot{\delta}_{\max} \\ & \quad \beta_{\min} \leq \beta \leq \beta_{\max} \\ & \quad \kappa_{\min} \leq \kappa \leq \kappa_{\max} \\ & \quad \alpha_{\min} \leq \alpha_f, \alpha_r \leq \alpha_{\max} \\ & \quad 0 \leq V \leq 1.1 \cdot V_{des} \end{aligned} \quad (17)$$

After solving (17) in each control cycle, a series of control input increments, $\Delta\psi_t^*$, in the control time domain, N_c , can be obtained by (18):

$$\Delta\psi_t^* = [\Delta u_t^*, \Delta u_{t+1}^*, \dots, \Delta u_{t+N_c-1}^*]^T \quad (18)$$

When the first element of the control sequence is applied to the system as the actual control increment, it can be calculated by (19):

$$\Delta\chi(t) = \Delta\chi(t-1) + \Delta\chi_t^* \quad (19)$$

After entering the next control cycle, repeat the above process, so as to track the trajectory through cyclic iteration.

III. SIMULATION EXPERIMENT

By building CarSim, Matlab / Simulink joint simulation platform, it is mainly to verify that the solution speed of the model controller after optimizing the objective function is faster.

The structure diagram of the simulation experiment is shown in Fig.2. The S-function in the figure is the implementation function of local path planning and trajectory tracking.

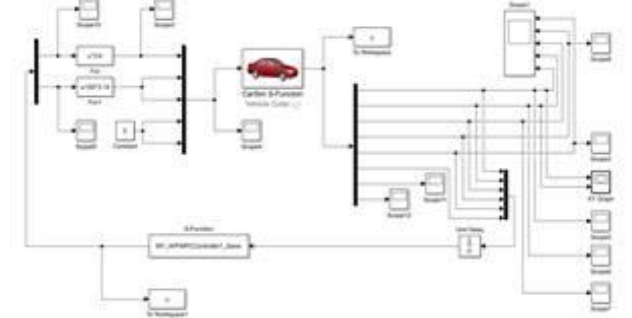


Figure 2. Structure of simulation experiment

In the experiment, eight simulation environments were designed according to the dynamic joining or leaving environment of environment vehicles, the existence of straddlable and unstraddlable obstacles on the lane, and whether the road was straight or curved.

Tab.III compares the time to solve the target solution before and after the improvement of MPC.

TABLE III. COMPARISON OF TIME CONSUMPTION BEFORE AND AFTER IMPROVING THE OBJECTIVE FUNCTION

Scenarios	Time consuming(S)	
	<i>Before</i>	<i>After</i>
Scenario 1	9.3	7.2
Scenario 2	7.8	6.2
Scenario 3	5.3	4.2
Scenario 4	5.3	4.1
Scenario 5	6.9	5.5
Scenario 6	4.2	3.4
Scenario 7	10.1	8.1
Scenario 8	9.9	7.9

It can be seen from the above table conclusion that by introducing APF and adjustment factor, the MPC controller improved by linearizing the objective function takes less time to solve the optimal path, which verifies that the solution speed of the optimized MPC is faster. At the same time, the experimental results also verify that the improved controller is more accurate in path planning and tracking, and the planned path is better.

IV. CONCLUSION

The optimized model controller of APF and MPC designed in this paper verifies that the unmanned vehicle can ensure the speed and tracking accuracy in the scene with interference vehicles and obstacles, and can realize

the following, obstacle avoidance, lane changing and overtaking according to the road conditions. The disadvantage is that the driving environment of unmanned vehicles in this paper is a structured road, but for complex road scenes, the influencing factors in the driving environment need to be further considered.

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