

Fungal Growth Kinetics Model Based on Carbon Cycle

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Abstract—Fungi play a critical role in the entire carbon cycle. In this article, the process of fungi participating in the carbon cycle was analyzed with differential equations, and a fungal growth kinetic model was set up through mechanism analysis. The results reflect that the optimum temperature required for different strains to reach the stable point is completely different, and the advantages and disadvantages of the growth of each strain under different temperature conditions are different. The model reveals the process of decomposing ground litter and wood fiber through its life activities in the presence of fungi.

Keywords—Fungi; Carbon cycle; Growth kinetic model

I. INTRODUCTION

Fungus is a kind of eukaryote, which mainly uses organic matter produced by other organisms as a carbon source for proliferation. Because of this, it plays an irreplaceable role in both the local carbon flow and the overall carbon cycle^[1].

Existing research has been able to determine the relationship between the fungal decomposition rate of litter and wood fiber on the ground and some of its characteristics, and can quantify it through experimental data^[2]; however, research on the dynamic characteristics of fungi decomposing organic matter and multi-species interaction mechanism^[3] is lacking. The former problem will be effectively solved in this article.

If a single fungal colony is simplified into a population, and a population growth model (usually a logistic model) is established on this basis, it can indeed reflect the process of fungi decomposing organic matter relatively effectively. However, this type of model assumes the simplest linear relationship between population growth rate and substrate decomposition rate, and it is difficult to accurately describe the dynamic characteristics of the decomposition process^[4]. So a different approach is taken which use: the idea of carbon cycle to describe the fungus' decomposition process of ground litter and wood fiber through the flow of carbon element. and thus the fungal growth kinetic model can be built up.

The process of carbon transfer from the ground litter to the fungus for metabolism can be summarized as follows: in the first step, lignin is decomposed by fungi, and carbon is transferred from the ground litter to the fungal community. Based on the Monod equation, a differential equation for the decomposition of the substrate is established^[5]. In this process, the falling rate of herbaceous plants is considered to be a constant, in order to quantitatively calculate the rate of increase of ground litter; in the second step, the fungal community uses the absorbed carbon to multiply and proliferate, and produce

metabolites. On this basis, the growth kinetic equation of the fungus was established. Through the above two steps, the growth kinetic model of a single strain is established. Then, the analytical method was adopted to analyze the stability of the differential equations and solve them numerically. Finally, the stable relationship between litter and colony carbon content was obtained^[6]. In addition, the stable carbon content of each strain under different temperatures and water potentials has also been analyzed to illustrate the advantages and disadvantages of each strain under different environments. Furthermore, the carbon content trend of various strains in different regional environments can also be predicted^[7]. Finally, the stability analysis of the metabolic rate of fungi is carried out, and the change of the stable point of the equations is discussed when the metabolic rate fluctuates within a certain range.

II. THE GROWTH KINETICS MODEL OF SINGLE SPECIES

The complete carbon cycle process in nature is very complicated. Therefore, in this article, the complete process is decomposed, and only the transfer of carbon from the ground litter to the fungus for metabolism is considered. In this segment, the flow of carbon has gone through two processes: firstly, lignin is decomposed by fungi, and carbon is transferred from the ground litter to the fungal community; then, the fungal community uses the absorbed carbon to multiply and proliferate, and produce metabolites.

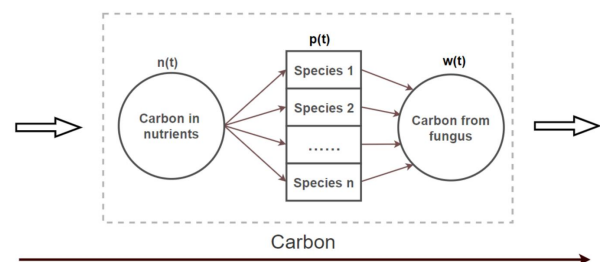


Figure 1. Schematic diagram of carbon flow

Based on the analysis of the mechanism of these two processes, a dynamic equation set consisting of the decomposition equation of the substrate (ground litter) and the improved kinetic equation of cell growth was established. Finally, the stability analysis of the equations and numerical methods are used to study the litter consumption and colony derivation trends under different parameters (temperature, water resistance), so as to study

the degradation effect of different colonies on the litter under various environmental variables.

A. Kinetic equation of single species' growth

The change of total carbon content in the species is composed with two parts: on the one hand, the fungal community constantly absorbs nutrients from the environment and increases; on the other hand, the death of fungi constantly occurs in the community. In the natural environment, it can be assumed that the ground litter is increasing at a constant rate, so the carbon content of litter that can be used by fungi is sufficient. Firstly, a model based on Monod equation is established^[5]. In the absence of inhibitors in the medium, the relationship between the specific growth rate of cells and the limiting matrix concentration can be expressed as follows:

$$\mu = \mu_{\max} \frac{n}{n+k}$$

In the above equation, μ stands for the growth rate ratio, μ_{\max} stands for the maximum specific growth rate, n stands for the limiting substrate concentration, k stands for the saturation constant, of which the value is the limiting substrate concentration when the specific growth rate is half of the maximum specific growth rate.

Based on Monod equation and considering the effects of fungal reproduction and death, a single species growth kinetics equation is established:

$$\frac{dp(t)}{dt} = \lambda \frac{n}{n+k} p(t) - \mu p(t)$$

In the above formula, $p(t)$ means carbon content of the colony; λ means the specific growth rate of the colony; k means the saturation constant and μ means the mortality of the fungal community per unit time.

B. The decomposition kinetic equation of the ground litter

As for the ground litter, it can be considered that part of the carbon is absorbed by fungi for proliferation and reproduction, and the other part is decomposed by fungi; at the same time, the ground litter is assumed to be increasing at a constant rate. Therefore, the kinetic equation of litter decomposition was established:

$$\frac{dn(t)}{dt} = -\lambda \frac{n}{n+k} p(t) - Wp(t) + c$$

In the above equation, W represents the metabolic rate of colony, specifically refers to the decomposition rate of substrate nutrients by enzymes produced in colony; c represents the the average rate of herbage withering.

Analyzing the above equation, $\frac{dn(t)}{dt}$ stands for the instantaneous change rate of the ground litter carbon content; $\lambda \frac{n}{n+k} p(t)$ stands for the amount of carbon absorbed by the colony in unit time, which will lead to the decrease of litter carbon content; $Wp(t)$ stands for the consumption of litter carbon by colony metabolism, which will further lead to the decrease of litter carbon content. c was used to represent the average rate of herbage withering,

which will lead to the increase of the ground litter carbon content.

C. The decomposition kinetic equation of the ground litter

Combined with the above analysis, a complete set of single species' growth kinetics equations is obtained as follows:

$$\begin{cases} \frac{dn(t)}{dt} = -\lambda \frac{n}{n+k} p(t) - Wp(t) + c \\ \frac{dp(t)}{dt} = \lambda \frac{n}{n+k} p(t) - \mu p(t) \end{cases}$$

In the above equations, the first equation is used to measure the change of carbon content of fungal community, and the second equation is used to measure the change of carbon content of ground litter. By calculating the stable point of the second-order differential equations, the growth state and litter decomposition efficiency of a single fungal community in the natural environment can be studied.

D. Stability analysis of growth kinetics equations

To study whether the dynamic system is stable through the Kinetic equations, stability analysis is very crucial. For a system of second order differential equations,

when $\frac{df(n^*, p^*)}{dn} + \frac{dg(n^*, p^*)}{dp} < 0$ and $|J| > 0$, the

corresponding equilibrium point can be obtained as (n^*, p^*) , which is stable. Through calculation, it can be concluded that:

$$\begin{cases} \frac{df(n^*, p^*)}{dn} + \frac{dg(n^*, p^*)}{dp} = -\frac{(\lambda - \mu)^2 c}{\lambda k (\mu + \mu)} < 0 \\ |J| = \frac{(\mu + w)(\lambda - \mu)^2 c}{\lambda k (\mu + w)} > 0 \end{cases}$$

Based on the above results, the equilibrium point can be considered stable

III. THE SETTLEMENT OF THE MODEL

The data on the relationship between the growth rate of different strains and the ambient temperature are found in Literature [7], which is as follows:

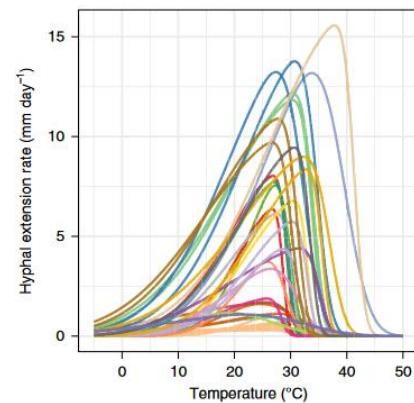


Figure 2. Curve of mycelial elongation with temperature

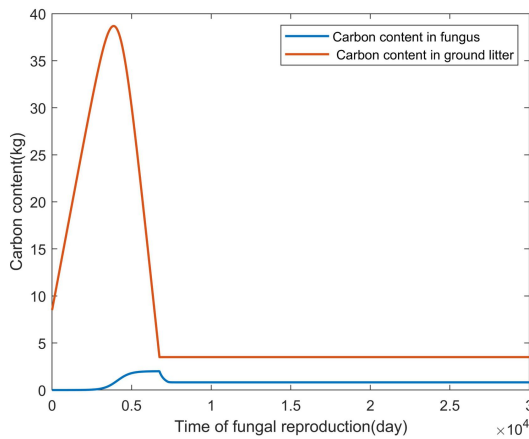
From the above images, with the increase of temperature and moisture, the growth rate of most species of fungi increase at first and then decrease. The reasons can be summarized as follows: before the temperature T reaches the optimal growth temperature T_0 , the growth rate λ increases with the increase of T , and reaches the peak at $T = T_0$, and then decreases with the increase of T until T reaches the maximum temperature T_{\max} , $\lambda = 0$, when the fungi is dead. Besides, due to different degrees of environmental adaptation to different fungi species, the suitable temperature ranges for fungi species also show great differences.

On the basis of establishing and analyzing the above equations, the species **a.gall.s** is chosen as an example. Fixing the saturation constant k , fungal metabolic rate w , the mortality rate of fungi μ , the temperature T , the water potential m , the dropping rate c , the initial conditions p_0, n_0 on a specific and reasonable value, as are shown in the table below:

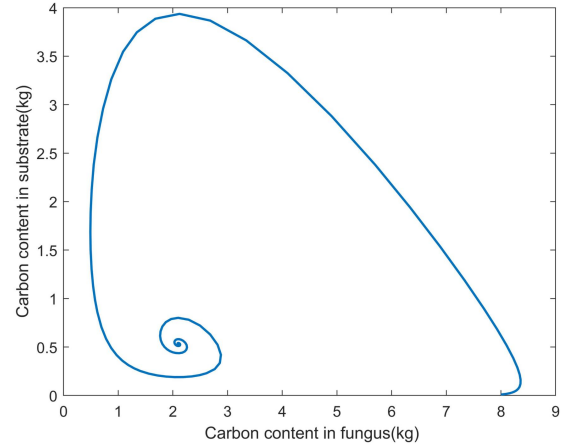
TABLE I. VALUES OF PARAMETERS AND CORRESPONDING UNITS

Parameter	Values	Unit
k	10^{-3}	—
w	0.01	—
μ	0.01	—
T	5.87	$^{\circ}\text{C}$
m	-0.43	MPa
c	0.04	kg/day
p_0	0.001	kg
n_0	5	kg

According to the cited data, it is available that the growth rate of this species under this initial condition is: $\lambda = 0.02$. To settle out the model, numerical solution was used and Growth curves of fungi as well as substrate (ground litter) consumption curves (5000 days reproduction of fungi) was drafted as follows:



(a) Trends of litter and fungi



(b) phase diagram

Figure 3. Results of the model

It is known from the figure that when the fungal reproduction number was 7320, the carbon content p of the colonies gradually leveled off, implying that the fungal growth and reproduction reached an equilibrium point at 7320 days, when the fungal birth rate and mortality rate were leveled off. At this time, the decreasing trend of the carbon content in the litter also tended to level off. As a result, the conclusion was obtained: the balance point of fungal growth and the balance point of litter consumption are the same with each other, which is also in accordance with the actual situation.

A. The impacts of different temperatures on the same species

When keeping other variables and parameters in the environment unchanged, and only changing the ambient temperature, the time for the same strain to reach a stable point is very different. In order to explore the specific influence of temperature on the decomposition rate of fungi, a representative fungus **a.gall.s** was selected as the research object. 5 sets of data on the relationship between the number of generations required for the fungus to reach a stable point and the temperature are sorted out, importing the data into MATLAB and drawing the image. The result is shown in Figure 4.

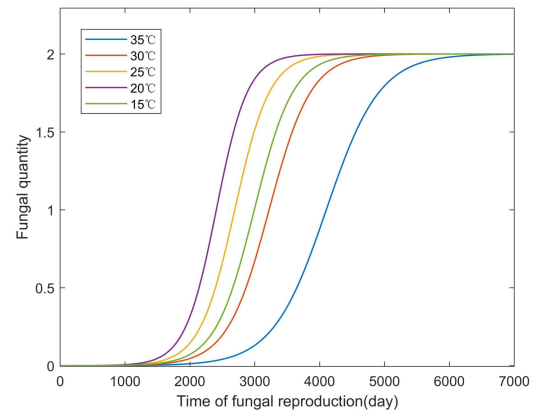
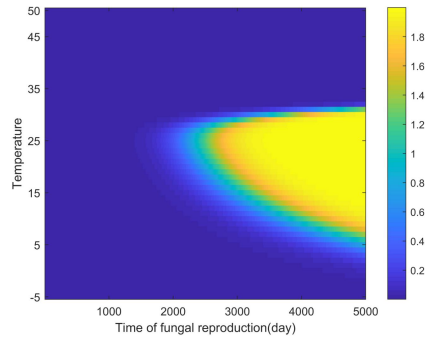


Figure 4. The relationship between fungal growth and temperature

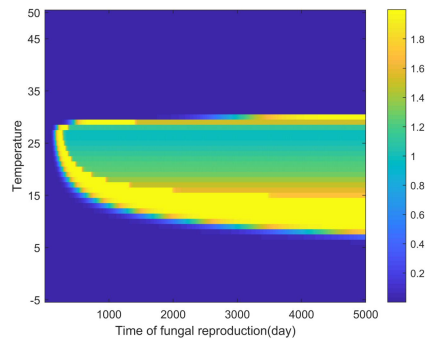
Because the overall activity and growth rate of the fungus is better between 15°C and 35°C, temperature in this interval are chosen to study the changes in fungal carbon content. Analyzing the above figure, it can be found that under different ambient temperatures T , the time for fungus **a.gall.s** to grow and reach the environmental capacity is different, and the required time decreases first and then increases as the ambient temperature T increases. After reaching a stable point for a period of time, the nutrients are consumed, and the fungal weight drops rapidly.

B. The influence of temperature changes on different species

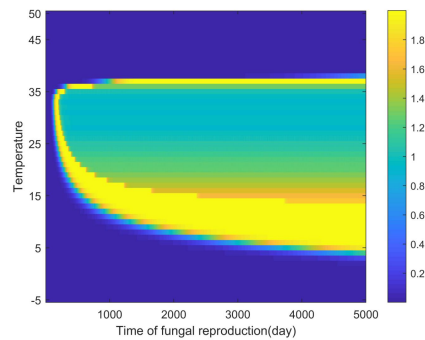
To explore the effects of temperature changes on different fungi, 3 representative species (fungus **a.gall.s**, fungus **I.carib.s**, fungus **s.comm.n**) were selected for analysis and the heat map is drafted to show the time required for the growth of each species to reach equilibrium at different temperatures, as shown below:



(a) Heat map of fungus **a.gall.s**



(b) Heat map of fungus **I.carib.s**



(c) Heat map of fungus **s.comm.n**

Figure 5. The relationship between fungal growth and temperature

The dark blue area indicates the period when the overall colony mass is small and the yellow area indicates the state after the colony has grown to a stable point. After the nutrients are gradually consumed, the total colony mass drops rapidly from the stable point. Observing the stable points of different strains at different temperatures, the most suitable temperature corresponding to the fastest time for each strain to reach the stable point can be found. Therefore, the optimum temperature for different strains is different

C. Summary

After analyzing the temperature changes on the fungus **a.gall.s**, fungus **I.carib.s** and fungus **s.comm.n**, the following conclusions are obtained: The optimum temperature for different fungus to reach a stable point is completely different, and there are differences in the advantages and disadvantages of the growth of each fungus under different temperature conditions.

IV. CONCLUDING REMARKS

Based on the carbon cycle, this paper discusses the fungal decomposition process of ground litter and establishes the corresponding differential equation model. The result of stability analysis proves that this model has excellent properties. Furthermore, this article discusses the influence of external environment temperature on the growth of fungi and the complex relationship between them, which is innovative and practical.

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