

## Research on Multi-Center Route Planning based on improved Ant Colony algorithm

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*Vehicle Routing Problem (VRP) is a hot issue in dynamic programming. It has a wide range of applications in the real economy and society. That is, a distribution center has several trucks full of goods, and trucks distribute goods to many demand points of the distribution center, requiring the optimization of the lowest total cost. In this paper, the improved ant colony algorithm is used to solve the multi-center VRP problem.*

**Keywords-** Ant colony algorithm; simulated annealing algorithm; multi-center; VRP; K-means

### I. Introduction

Heuristic algorithm is the most effective method to solve the NP complete problem. It gradually shows strong, distributed, self-organizing and other advantages in a large number of scientific research activities and engineering applications, and ant colony algorithm is more heuristic The leader in the algorithm. This paper explores the application of ant colony algorithm in vehicle routing problems, and proposes a new ant colony algorithm suitable for various types of vehicle routing problems, and has achieved certain research results.

### II. Multi-center to single center

First of all, the core process and target of the K-Means algorithm are relatively simple. We have an initial data set, and there are many scattered points in the two-dimensional coordinate system plane. Our goal is to divide the initial data set into multiple smaller subsets, each point belongs to and belongs to only one subset, and the union of all subsets is equal to the initial data set. The multi-center vehicle routing problem studied in this paper is divided into single-center problems by K-means clustering. Each central point is responsible for the supply of its closely associated strongholds.

We use the K-means algorithm for programming, and plan to divide randomly generated points into four categories, and represent the resulting scatter plot of the clustering results as Figure 1.

### III. VRP problem

#### A. VRP problem description

In the previous chapter, we transformed the multi-center VRP problem into a single-center VRP problem. In the VRP problem, the coordinates of each office and the coordinates of the distribution center are known. Among

them, the starting point of all vehicles is the distribution center. The number of points is  $n$ , which means each distribution center, the number of vehicles is  $m$ , it has is expressed by the quality of these vehicles is the same, and the quality of the goods they can carry is also the same,  $D$  expressed by the vehicle from the office to travel. The cost of consumption from  $i$  to  $j$  is expressed in terms of  $c_{ij}$ , the goods required by the period are expressed in terms of  $q_i$ , and the distance from the office to travel  $i$  to the need to travel  $j$  is expressed in terms of  $d_{ij}$ , we will describe its corresponding VBP mathematical model.

The objective function is as follows

$$\min \sum_{k \in m} \sum_{i \in n} \sum_{j \in n} c_{ij} d_{ij} x_{ijk} \quad (1)$$

Restrictions:

$$\sum_{k \in m} y_{ik} = 1, 2, \dots, n; \quad (2)$$

$$\sum_{i=0} \sum_{k=1} x_{ijk} = 1, 2, \dots, n; \quad (3)$$

$$\sum_{j=0} \sum_{k=1} x_{ijk} = 1, 2, \dots, n; \quad (4)$$

$$\sum_{i=1}^n y_{ik} q_i = D; \quad (5)$$

$$\sum_{i=1}^n x_{i0k} - \sum_{j=1}^n x_{0jk} = 0; \quad (6)$$

$$y_{ik} = \begin{cases} 1, & \text{the car } k \text{ for point } i \\ 0, & \text{else} \end{cases} \quad (7)$$

$$x_{ijk} = \begin{cases} 1, & \text{the car } k \text{ from point } i \text{ to point } j \\ 0, & \text{else} \end{cases} \quad (8)$$

In the above formulas, formula (1) represents the minimum value of the transportation cost; it is related to the transportation distance and increases with the increase of the distance; equations (2)-(4) stipulate that for each office, There is only one vehicle for delivery of goods; formula (5) means that the amount of each vehicle is equal to the maximum load of the vehicle in this article; the meaning of formula (6) is that the truck carrying the goods completes the transportation and returns to the circuit of the starting point. Excluding the time factor, our cost only considers the distance, and the concept of truck speed will not be introduced.

#### B. Principle of ant colony algorithm

The ant colony algorithm generally has the following two optimization directions:

(1) Solution space probability functions, such as state transition probability function and information update function.

(2) Basic behavior rules, that is, regular behavior evolution simulates the behavior of ant colony foraging and clustering.

The ant colony algorithm includes two main stages: ant colony path construction and pheromone update.

The set of  $N$  sites is  $C_n = \{c_1, c_2, c_3, \dots, c_n\}$ ,  $d_{ij}(i, j = 1, 2, \dots, n)$  set to have a path connection between any two sites. The path calculation here uses Euclidean distance, that is  $A(x_1, y_1), B(x_2, y_2)$ , for two-dimensional coordinates, there is

$$d_{AB} = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} \quad (9)$$

Then at that point, a distance matrix can be constructed:

$$\begin{bmatrix} d_{11} & \dots & d_{1n} \\ \vdots & \ddots & \vdots \\ d_{n1} & \dots & d_{nn} \end{bmatrix} \quad (10)$$

Route construction: Each truck takes the center point of the distribution center as the starting point, and remembers a route access sequence. This sequence includes the sites that are sequentially passed during the delivery process of the delivery truck. When going to the next city, the choice of the city is random.

Random selection is based on the following rules:

$K$  represents the number of the truck, and  $R^k$  records the serial numbers of all the stations visited by truck  $k$ . The current station where truck  $k$  is located is  $i$ , and the probability that  $j$  is the next visited city of truck  $k$  is:

$$P_k(i, j) = \begin{cases} \frac{[\tau(i, j)]^\alpha [\rho(i, j)]^\beta}{\sum_{u \in J_k(i)} [\tau(i, u)]^\alpha [\rho(i, u)]^\beta}, & j \in J_k(i) \\ 0, & \text{else} \end{cases} \quad (11)$$

$J_k(i)$  Represents a collection of cities that can be reached directly from city  $i$  and no longer access the record

$\rho(i, j)$  Is a heuristic message, usually calculated by  $\rho(i, j) = 1/d_{ij}$   $\tau(i, j)$  show the Pheromone quantity of  $(i, j)$ . Through the above formula, we can draw the following conclusion. In all paths, the easier it is for ants to choose the one with the shorter distance and more pheromones. In the formula, the weight of the influence of the heuristic information on the pheromone concentration in the whole process is expressed by assuming the sum of the two parameters ( $\alpha$  and  $\beta$ ) in advance. When  $\alpha = 0$ , the result obtained by this algorithm becomes the truck most likely to choose to leave The nearest city, that is, the random greedy calculation method; similarly, when  $\beta = 0$ , only pheromones played a role in the choice of ants' behavior. At this time, the convergence rate of the entire algorithm is very fast, and the final optimal path is not the actual one we need. The optimal path, that is, the performance of the algorithm appears to be very poor. The empirical value shows that it is more appropriate when  $\alpha = 1, \beta = 2 \sim 5$

In the algorithm implementation, this block uses the famous roulette algorithm.

Roulette selection algorithm:

The roulette selection algorithm is based on the ratio of the calculated access probability of each round to the sum of all rounds

Secondly, suppose a roulette with  $N$  sectors, each sector corresponds to a site, the size of the sector is proportional to the value of the  $P_i$  corresponding site:

$$S = \partial P_i \quad (12)$$

The probability that an individual office is selected is:

$$P(x_i) = \frac{f(x_i)}{\sum_{j=1}^N f(x_j)} \quad (13)$$

Once the roulette wheel rotates, the pointer on the roulette wheel will point to a specific sector after the rotation, and the corresponding point is a point in the path selected by the ant. After the roulette wheel turns  $N$  times in sequence, we can A path vector consisting of  $N$  points is obtained.

Cumulative probability:

Cumulative probability expresses the probability of each individual using line segments of different lengths. These line segments are combined into a straight line with a length of 1 (sum of individual probabilities). In this straight line, the longest line segment of a certain segment represents The greater the probability that the individual is selected. Its mechanism is:

(1) Arbitrarily select an arrangement sequence of all individuals (this sequence can be arranged randomly, because the length between a certain line segment represents the selection probability of a certain body)

(2) The cumulative probability of any individual is the cumulative sum of the previous data corresponding to that individual.

The cumulative probability formula of an individual is as follows:

$$Q(x_i) = \sum_{k=1}^i p(x_k) \quad (14)$$

According to the roulette selection algorithm, the  $P$  value of the better positions is larger, the longer the corresponding line segment, the greater the probability of being selected

Pheromone update:

The initial pheromone  $\tau_0 = m/C^m$ ,  $m$  is the number of trucks, and  $C^m$  is the length of the path constructed by the greedy algorithm.

The process of the greedy algorithm: starting from the distribution center, each time the point with the shortest path is selected from the currently reachable points (unvisited and the remaining load on the truck is greater than the demand of this site), and then transferred to that point, And then continue to execute the algorithm. When it is impossible to choose, the truck returns to the distribution center, and the algorithm ends.

Pheromone update step: Every time a round is completed, the pheromone present on each path will be reduced. For this, after each round, we multiply a constant  $\rho$  less than 1 on the basis of the previous pheromone. After this, each truck will release the pheromone according to the distance of the path established by itself and when it passes the path. For the truck, the delivery path established by the truck is short enough that it can release the pheromone when passing That is enough, the higher the frequency of any path, the higher the pheromone concentration.

$$\tau(i, j) = (1 - \rho) * \tau(i, j) + \sum_{k=1}^m \Delta \tau_k(i, j) \quad (15)$$

$$\Delta\tau_k(i,j) = \begin{cases} (C_k)^{-1}, & (i,j) \in R^k \\ 0, & \text{else} \end{cases} \quad (16)$$

In the formula,  $\rho$  is used to indicate the degree of evaporation of the pheromone in the path with time. The range of values is:  $0 < \rho \leq 1$ , the number of  $\Delta\tau_k(i,j)$  means pheromone released by the  $k$ th ant when passing a certain path,  $C_k$  referring to the distance of the constructed path. The value of  $R^k$  is equal to the distance of all the edges in the sum.

After assigning a delivery route to each truck, we have a delivery sequence. Of course, this solution is relatively superior, in order to make this solution closer to the optimal solution, we need to introduce simulated annealing algorithm to improve.

### C. Improved ant colony algorithm to solve VRP problem

Simulated annealing algorithm solution process:

After we get the distribution route of each truck, we use this sequence as the initial temperature solution.

In the vector of  $\{1,2,3,N\}$ , each fixed starting point and its ending point are cyclically arranged, and the set they form is the solution space.

$S = \{(\pi_1, \dots, \pi_N) | \pi_1=1, (\pi_2, \dots, \pi_N) \text{ is circular array of } \{2,3,4,N\}, \pi_N=N\}$

$N$  represents the number of sites in a single center. Any number in the solution space  $S$  represents a site, and any combination of these sites represents the solution of a path.

(1) Objective function. The objective function is the path length of the goods to all offices.

$$\min f(\pi_1, \pi_2, \dots, \pi_N) = \sum_{i=1}^N d\pi_i \pi_{i+1} \quad (17)$$

(2) The generation of a new solution, let the solution obtained by the ant colony algorithm be  $S_1$

i. The traversal position of two sites in any switching path

ii. Arbitrarily insert a certain position behind any office,

(3) Cost function difference, calculate the difference between the two paths before and after transformation

$$\Delta f = f_{\text{after}} - f_{\text{before}}$$

(4) Acceptance criteria

$$P = \begin{cases} 1, & \Delta f < 0 \\ \exp\left(-\frac{\Delta f}{T}\right), & \Delta f \geq 0 \end{cases} \quad (18)$$

(5) To decrease the temperature, multiply the original temperature by a constant less than 1 to indicate the decrease in temperature, that is,  $\alpha T$  to indicate the new temperature;

(6) Extract a temperature value  $e$  that needs to be lowered for the algorithm setting. When the temperature drops to the set value, it means that the annealing algorithm ends.

The number of iterations in the entire calculation process is  $L$ .

### D. Algorithm and programming simulation

Using the data set of the previous chapter, we use Visual Studio 2015 version on the Window 10 operating

system platform to program the traditional ant colony algorithm and improved ant colony algorithm.

Based on the previous data set, we used the Visual Studio 2015 version of the Windows 10 operating system to simulate the ant colony algorithm. We selected 1000 iterations as the convergence condition; the shortest path of each distribution center was as follows.

Table 3.1 The shortest path of each distribution center before optimization

delivery center 1		896.288
delivery center 2		720.632
delivery center 3		803.448
delivery center 4		1093.18

The shortest path of the total optimal solution is: 3513.549707; the corresponding path diagram is as Figure 2:

After using simulated annealing algorithm to optimize, the shortest path of each distribution center is as follows.

Table 3.2 The shortest path of each distribution center after optimization

delivery center 1	5	778.747
delivery center 2	6	714.184
delivery center 3	5	686.467
delivery center 4	7	886.095

The shortest path of the total optimal solution is 3065.439003; the corresponding path diagram is as Figure 3:

### E. Summary of this chapter

This chapter first introduces the VRP problem and describes it with a mathematical model; then separately introduces the ant colony algorithm in detail and makes improvements, respectively, using two algorithms to simulate the randomly generated data set, and obtains the improved algorithm. The path is significantly smaller than before the improvement.

## IV. Summary and outlook

### A. Full text summary

Starting from the origin of the ant colony algorithm, this paper has made great improvements to the algorithm, and has studied the problem of vehicle routing optimization layer by layer. The main innovations of this article are as follows:

(1) Use the K-means clustering algorithm to transform the multi-center problem into a single-center problem.

(2) In the VRP problem without a time window, the roulette selection algorithm is used to optimize the parameter settings, and a path selection mechanism based on the principle of greedy algorithm is created.

(3) In the VRPTW problem, the scheduling algorithm in the CPU—a high response priority ratio scheduling algorithm is used to optimize the parameters, and a pheromone update method—reasonable path method is creatively proposed.

### B. Future Outlook

On the basis of this paper, the following problems in this field are worthy of further study:

(1) In the follow-up scientific research work, the mathematical model of the vehicle routing problem needs to be further improved. By establishing a more reasonable mathematical model, the vehicle routing problem will also be more practical.

(2) Considering the constraint conditions that are more in line with the actual situation, the ant colony algorithm is more widely applied to new vehicle routing problems. In reality, there are often comprehensive and complex constraints. If the vehicle routing problem can be considered more widely, no matter what dilemma it faces, a theoretical basis can be found.

(3) As a probabilistic algorithm, the proof of the convergence speed and complexity of the ant colony algorithm is still in the vacancy stage. Being able to learn and develop the ant colony algorithm in a more scientific way will make it more efficient and more effective in handling vehicle routing problems. Targeted.

(4) Existing ant colony algorithm has a huge cost when solving large-scale vehicle routing problems. Faced

with the shortcomings of ant colony algorithm, it is necessary to make use of the advantages of other heuristic algorithms to make up, and construct a new ant colony algorithm and other algorithms Mixed strategy. [13]

In summary, in theory research and economic and social practical applications, the ant colony algorithm has great value and significance in the vehicle routing problem.

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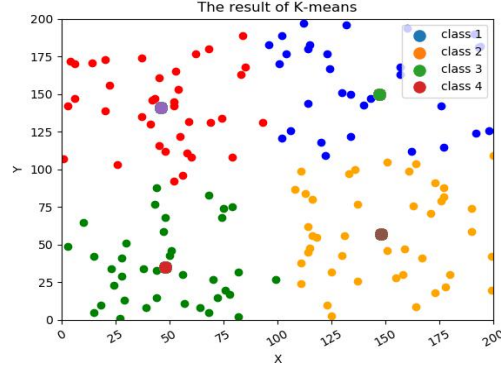


Figure 1 Scatterplot of clustering results

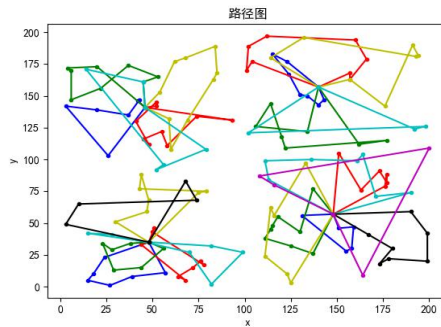


Figure 2 Roadmap before optimization

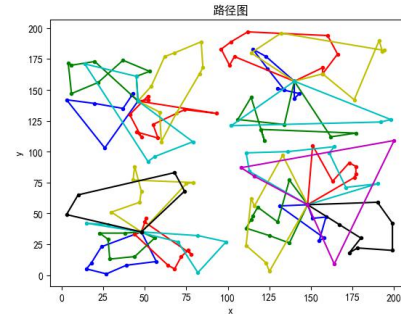


Figure 3 After path optimization