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Finite-time Combination Projective Synchronization for Memristive Systems

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***Abstract*— In this article, based on the sliding-mode controller, the finite-time combination synchronization of three memristive FitzHugh-Nagumo(FHN) systems is carried out. This controller drives the system to achieve combination synchronization faster. The synchronization characteristics are investigated by the error curve of the combination systems, the curve of the sliding mode switching surface, and relationship curves of system variables, respectively. Numerical results are certified the correctness of the controller.**

Keywords-FitzHugh–Nagumo system; Memristor-based chaotic systems; Finite-time synchronization

I. INTRODUCTION

In 1990, since the original work[1], chaotic synchronization has attracted extensive attention from scholars due to the good application of memristive chaotic system synchronization in information processing, secure communication and image encryption [2-4]. Several control strategies to achieve synchronization are investigated, such as adaptive control[5], feedback control[6], sliding mode control[7]. Many studies related to the memristive chaotic system have been conducted [8-12]. Based on the combined synchronization and projective synchronization models, the combined- combined synchronization of multiple different memristive systems is realized through sliding mode control. [10]. A non-autonomous chaotic FHN neuron system based on memristance was constructed, and the synchronization conditions of two unidirectional and bidirectional memristive FitzHugh-Nagumo neuron circuits were studied[11]. A five-dimensional hyperchaotic four-wing system based on memristor was built and its synchronization was realized[12]. However, the above-mentioned research only reports on a master-slave system, and further study on the synchronization of two or more memristive systems is needed. In addition, in practice, it is hoped that the system can be synchronized quickly. Therefore, it is meaningful to study the finite-time combination projective synchronization of multiple memristive systems.

Based on the method of sliding mode control, a new sliding-mode controller is designed to realize the projective synchronization of three memristive FHN systems with unknown interference. The controller is simple in structure and can make systems quickly realize combination projective synchronization.

II. DESIGN OF SLIDING MODE CONTROLLER

The two driving systems and the response system are written as follows:

$$\dot{x}=A_1x+f_1(x)+D_1(t) \tag{1}$$

$$\dot{y}=A_2y+f_2(y)+D_2(t) \tag{2}$$

and

$$\dot{z}=A_3z+f_3(z)+D_3(t)+u(t) \tag{3}$$

where $x=[x_1,x_2...x_n]^T$, $y=[y_1,y_2...y_n]^T$ and $z=[z_1,z_2...z_n]^T$ are the state vectors of systems (1)-(3),respectively. $A_1=[A_{11},A_{12}...A_{1n}]^T, A_2=[A_{21},A_{22}...A_{2n}]^T, A_3=[A_{31},A_{32}...A_{3n}]^T$ are coefficient matrix, $f_1(x)=[f_{11}(x),f_{12}(x)...f_{1n}(x)]^T$, $f_2(y)=[f_{21}(y),f_{22}(y)...f_{2n}(y)]^T, f_3(z)=[f_{31}(z),f_{32}(z)...f_{3n}(z)]^T$ are continuous functions, $D_1(t)=[d_{11},d_{12}...d_{1n}]^T, D_2(t)=[d_{11},d_{12}...d_{1n}]^T, D_3(t)=[d_{31},d_{32}...d_{3n}]^T$ are the external disturbances, $u(t)=[u_1(t),u_2(t)...u_n(t)]^T$ is the vector of synchronous controllers. If there are three constant matrices $B,C,J\in R^{n\times n}$ and $J\neq 0$, a constant $T=T($, $e(0))>0$ such that $\lim\limits_{t\to T}\|Bx+Cy-Jz\|=0$,if $t\geq T$,the combination of the driving system (1) and (2) and the response system (3) realize the combination projective synchronization in a limited time.

Assume $B=I, C=I$ and $J=diag(J_1,J_2...J_n)$ .The synchronization error of the systems can be rewritten as:

$$\lim_{t\to T}\|e(t)\|=\|x+y-Jz\|=0 \tag{4}$$

From (1)-(3), the resulting error dynamics is described as follows:

$$\begin{aligned}\dot{e}&=\dot{x}+\dot{y}-J\dot{z}\\ &=A_1x+f_1(x)+D_1(t)+A_2y+f_2(y)+D_2(t)-\\ &\quad J\left[A_3z+f_3(z)+D_3(t)+u(t)\right]\\ &=A_1e+A_2e+(A_1J+A_2J-JA_3)z-A_1y-A_2x+f_1(x)+\\ &\quad f_2(y)-Jf_3(z)+D_1(t)+D_2(t)-JD_3(t)-Ju(t)\end{aligned} \tag{5}$$

Assumption 1. The external interferences are bounded, that is,

$$\|D_1(t)\|\leq\alpha,\|D_2(t)\|\leq\beta,\|JD_3(t)\|\leq\gamma \tag{6}$$

where α,β,γ are known constants.

Lemma 1. If there is a differential positive definite function $V(t)$ such that

$$\dot{V}(t)\leq -kV^{\theta}(t),\forall t\geq t_0,V(t_0)\geq 0 \tag{7}$$

where $k>0, 0<\theta<1$ are constants. Then function $V(t)$ satisfies

$$V^{1-\theta}(t)\leq V^{1-\theta}(t_0)-k(1-\theta)(t-t_0), t_0\leq t\leq t_1 \tag{8}$$

and

$$V(t)\equiv 0,\forall t\geq t_1 \tag{9}$$

with the settling time t_1 satisfying

$$t_1\leq t_0+\frac{V^{1-\theta}(t_0)}{k(1-\theta)} \tag{10}$$

Lemma 2(see [13]). Suppose $\tau_1,\tau_2...\tau_n$ and $0<\mu<1$ are real numbers, and this is the following inequality

$$\sum_{i=1}^{n}|\tau_i|^{\mu+1}\geq\left(\sum_{i=1}^{n}|\tau_i|^2\right)^{\frac{\mu+1}{2}} \tag{11}$$

Choose the following sliding surface:

$$s = \lambda e \tag{12}$$

where $\lambda = [\lambda_1 \lambda_2, ..., \lambda_n]$ is a constant vector.

In order to design the controller, we consider the following sliding mode switching surface

$$\dot{s}_i = -\rho_i \operatorname{sgn}(s_i)|s_i|^{\mu} - \delta_i \eta, i = 1,2,...,n \tag{13}$$

where $\eta = \frac{s_i}{|s_i| + \alpha}$ is a continuous function, $\operatorname{sgn}(\cdot)$ represents the sign function. $\rho_i > 0$ and $\delta_i > 0$ are gains. α and $0 < \mu < 1$ are constants.

The control input is given as follows:

$$\begin{aligned} u(t) = J^{-1}[(A_1 J + A_2 J - JA_3)z + f_1(x) + f_2(y) \\ - Jf_3(z) - A_1 y - A_2 x] - J^{-1} q \vartheta(t) \end{aligned} \tag{14}$$

where $q = [q_1, q_2 ... q_n]^T$ is a constant gain, and $\vartheta(t)$ is the control input, which satisfies the following formula.

$$\vartheta(t) = \begin{cases} \vartheta^+ & s \geq 0 \\ \vartheta^- & s \leq 0 \end{cases} \tag{15}$$

From (14) and (12), the error system is re-expressed as follows:

$$\dot{e} = A_1 e + A_2 e + q\vartheta(t) + D_1(t) + D_2(t) - JD_3(t) \tag{16}$$

From (12), (13) and (16), we get:

$$\begin{aligned} \vartheta(t) &= -q^{-1}(A_1 e + A_2 e + D_1(t) + D_2(t) - JD_3(t) - \dot{e}) \\ &= -q^{-1}(A_1 e + A_2 e + D_1(t) + D_2(t) - JD_3(t) - \lambda^{-1}\dot{s}) \\ &= -\lambda^{-1} q^{-1}(\lambda A_1 e + \lambda A_2 e + \lambda D_1(t) + \lambda D_2(t) - \lambda JD_3(t) \\ &\quad + \rho \operatorname{sgn}(s)|s|^{\mu} + \delta \frac{s}{|s| + \alpha}) \\ &= -(\lambda q)^{-1}(\lambda A_1 e + \lambda A_2 e + \lambda D_1(t) + \lambda D_2(t) - \lambda JD_3(t) \\ &\quad + \rho \operatorname{sgn}(s)|s|^{\mu} + \delta \frac{s}{|s| + \alpha}) \end{aligned} \tag{17}$$

Since the external disturbances $D_1(t), D_2(t)$ and $D_3(t)$ are unknown, the control law is rewritten as

$$\vartheta(t) = -(\lambda q)^{-1}(\lambda A_1 e + \lambda A_2 e + \rho \operatorname{sgn}(s)|s|^{\mu} + \delta \frac{s}{|s| + \alpha}) \tag{18}$$

Theorem 1. By using the control law in (18), the combination projective synchronization of the system (1)-(3) can be achieved under the premise of satisfying the condition (19).

$$\|\lambda\|(\alpha + \beta + \gamma) - \rho < 0 \tag{19}$$

Proof Lyapunov function is chosen as

$$V = \frac{1}{2}s^2 \tag{20}$$

and the derivative of V is worked out

$$\dot{V} = s\dot{s} \tag{21}$$

From (12), (5) and (21), we obtain

$$\begin{aligned} \dot{V} &= s\lambda\dot{e} \\ &= \lambda s(\dot{x} + \dot{y} - J\dot{z}) \\ &= \lambda s(A_1 e + A_2 e + (A_1 J + A_2 J - JA_3)z - A_1 y \\ &\quad - A_2 x + f_1(x) + f_2(y) - Jf_3(z) + D_1(t) \\ &\quad + D_2(t) - JD_3(t) - Ju(t)) \end{aligned} \tag{22}$$

and substituting the equations (16)-(18) into the equations (22), one gets

$$\begin{aligned} \dot{V} &= s\lambda\dot{e} \\ &= \lambda s(A_1 e + A_2 e + D_1(t) + D_2(t) - JD_3(t) \\ &\quad - q\lambda^{-1} q^{-1}(\lambda A_1 e + \lambda A_2 e + \rho \operatorname{sgn}(s)|s|^{\mu} + \delta \frac{s}{|s| + \alpha})) \\ &= \lambda s(D_1(t) + D_2(t) - JD_3(t)) - s \operatorname{sgn}(s)|s|^{\mu} - \delta \frac{s^2}{|s| + \alpha} \\ &= \lambda s(D_1(t) + D_2(t) - JD_3(t)) - \rho|s|^{\mu+1} - \delta \frac{s^2}{|s| + \alpha} \end{aligned} \tag{23}$$

By Assumption 1, we can get

$$\begin{aligned} \dot{V} &= s\lambda\dot{e} = \lambda s(D_1(t) + D_2(t) - JD_3(t)) - \rho|s|^{\mu+1} - \delta \frac{s^2}{|s| + \alpha} \\ &\leq \|\lambda\|\|s\|[\|D_1(t)\| + \|D_2(t)\| + \|JD_3(t)\|] - \rho|s|^{\mu+1} - \delta \frac{s^2}{|s| + \alpha} \\ &\leq [\|\lambda\|(\alpha + \beta + \gamma) - \rho]\|s\| - \rho|s|^{\mu} - \delta \frac{s^2}{|s| + \alpha} \\ &\leq -\rho|s|^{\mu} \end{aligned} \tag{24}$$

Through the lemma, we obtain

$$\dot{V}(t) \leq -2^{\frac{\mu}{2}} \rho V^{\frac{\mu}{2}} \tag{25}$$

The errors will tend to the set sliding surface in a limited time. Hence, the proof is completed.

$$T_1 \leq \frac{(V(t_0))^{\frac{2-\mu}{2}}}{2^{\frac{\mu-2}{2}} \rho(2-\mu)} \tag{26}$$

III. FINITE-TIME COMBINATION SYNCHRONIZATION OF MEMRISTIVE FITZHUGH-NAGUMO SYSTEMS

In order to prove the correctness of the designed controller, FitzHugh-Nagumo [14] based on memristor was selected as the research object for numerical simulation. Three memristive FitzHugh-Nagumo systems with different initial states were selected.

The two drive systems are shown as

$$\begin{cases} \dot{x}_1 = x_2 + ax_1 + (b/d)\ln\cosh(dx_1 - n_1) + 1.8\sin(\tau) \\ \quad + g_1 - (b/d)\ln\cosh(n_1) + d_{11} \\ \dot{x}_2 = -c(x_1 + x_2) + m_1 + d_{12} \end{cases} \tag{27}$$

and

$$\begin{cases} \dot{y}_1 = y_2 + ay_1 + (b/d)\ln\cosh(dy_1 - n_2) + 1.8\sin(\tau) \\ \quad + g_2 - (b/d)\ln\cosh(n_2) + d_{21} \\ \dot{y}_2 = -c(y_1 + y_2) + m_2 + d_{22} \end{cases} \tag{28}$$

The corresponding slave system is described as

$$\begin{cases} \dot{z}_1 = z_2 + az_1 + (b/d)\ln\cosh(dz_1 - n_3) + 1.8\sin(\tau) \\ \quad + g_3 - (b/d)\ln\cosh(n_3) + d_{31} + u_1(t) \\ \dot{z}_2 = -c(z_1 + z_2) + m_3 + d_{32} + u_2(t) \end{cases} \tag{29}$$

The system parameters and variables are set with $a = 0.5, b = 0.5, c = 1, d = 1$.

From Eq. (27), (28) and (29), one easily obtains

$$A_1 = \begin{bmatrix} 0.5 & 1 \\ -1 & -1 \end{bmatrix}, A_2 = \begin{bmatrix} 0.5 & 1 \\ -1 & -1 \end{bmatrix}, A_3 = \begin{bmatrix} 0.5 & 1 \\ -1 & -1 \end{bmatrix}$$

$$f_1(x)=\begin{bmatrix}0.5\ln\cosh(x_1-n_1)+g_1-0.5\ln\cosh n_1\\ m_1\end{bmatrix}$$
$$f_2(x)=\begin{bmatrix}0.5\ln\cosh(y_1-n_2)+g_2-0.5\ln\cosh n_2\\ m_2\end{bmatrix} \tag{30}$$
$$f_3(x)=\begin{bmatrix}0.5\ln\cosh(z_1-n_3)+g_3-0.5\ln\cosh n_3\\ m_3\end{bmatrix}$$
$$D_1=\begin{bmatrix}0.4\cos(10t)\\ -0.5\sin(20t)\end{bmatrix},D_2=\begin{bmatrix}-0.5\sin(30t)\\ 0.3\cos(20t)\end{bmatrix},D_3=\begin{bmatrix}0.4\sin(20t)\\ 0.2\sin(30t)\end{bmatrix}$$

Assuming $J=diag\{3,-2\}$, the error system is given as:

$$\begin{aligned}\dot{e}_1 &= 0.5e_1+e_2-5z_2+0.5\ln\cosh(x_1-n_1)\\ &\quad -0.5\ln\cosh(n_1)+0.5\ln\cosh(y_1-n_2)\\ &\quad -0.5\ln\cosh(n_2)+1.51\ln\cosh(n_3)\\ &\quad -1.5\ln\cosh(z_1-n_3)-1.8\sin(\tau)+g_1+g_2\\ &\quad -3g_3+d_{11}+d_{21}-3d_{31}+3u_1\\ \dot{e}_2 &= -e_1-e_2-5z_1+m_1+m_2+2m_3+d_{12}+d_{22}\\ &\quad +2d_{32}-2u_2\end{aligned} \tag{31}$$

And the control parameters are taken by $\lambda=[1,1]$, $q=[1,1]^T$, $\rho_1=3$ and $\delta_1=3$.

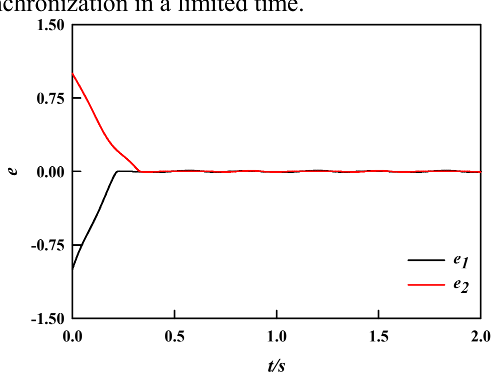
Thus, the result is obtained

$$\begin{aligned}\vartheta_1(t) &= 0.5e_1-1.5\frac{e_1}{|e_1+e_2|+0.01}-1.5\frac{e_2}{|e_1+e_2|+0.01}\\ &\quad -1.5\operatorname{sgn}(s)|e_1+e_2|^{\mu}\end{aligned} \tag{32}$$

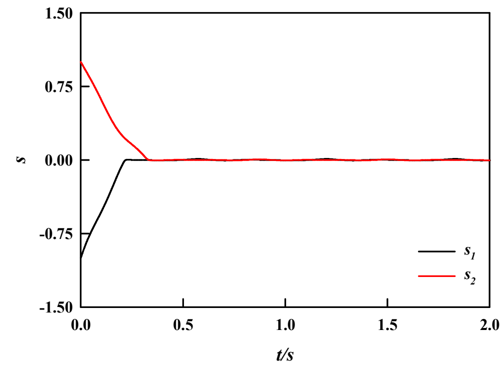
that is,

$$\vartheta_1(t)=\begin{cases}0.5e_1-1.5\dfrac{e_1}{|e_1+e_2|+0.01}-1.5\dfrac{e_2}{|e_1+e_2|+0.01}\\ -1.5|e_1+e_2|^{\mu} & s>0\\ 0.5e_1-1.5\dfrac{e_1}{|e_1+e_2|+0.01}-1.5\dfrac{e_2}{|e_1+e_2|+0.01}\\ +1.5|e_1+e_2|^{\mu} & s<0\end{cases} \tag{33}$$

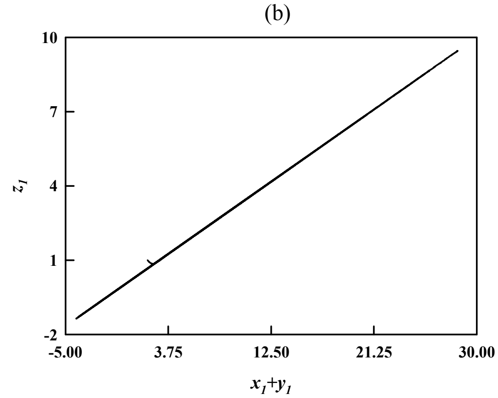
The initial states of system in (27)-(29) are set by $x(0)=[0,0], g_1=0.1, m_1=0, n_1=0, y(0)=[2,-1], g_2=0, m_2=0$ $n_2=-0.2$ and $z(0)=[1,1], g_3=0, m_3=0, n_3=0.3$. respectively. Controller parameter is set by $\mu=0.6$, $\alpha=[0.01,0.01]^T$, $\lambda=[1,1], q=[1,1]^T, \rho=[3,2]^T$ and $\delta=[3,2]^T$. Numerical results are plotted in Fig.1. From Fig.1, errors of the system can quickly approach zero and the systems (27)-(29) realize the combination projective synchronization in a limited time.



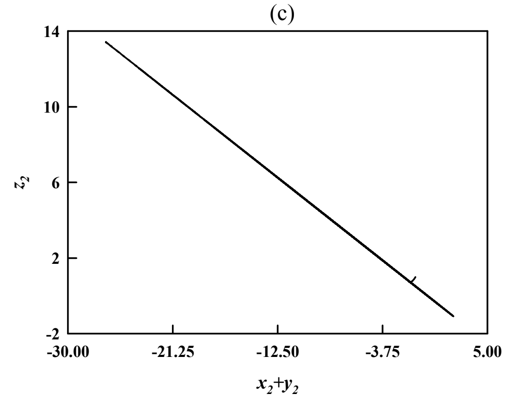
(a)



(b)



(c)



(d)

Fig. 1 Synchronization simulation diagram:(a)the error curves of synchronization, (b) The trajectory of sliding surface,(c) Relationship between state variables x_1+y_1 and z_1 ,(d) Relationship between state variables x_2+y_2 and z_2.

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