# Projective Synchronization for Heterogeneous Active Magnetic Controlled Memristor-based Chaotic Systems

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*Abstract*— **In this paper, a nonlinear synchronous controller with exponential function based on Lyapunov stability theory is designed to realize the projective synchronization of two active magnetic controlled memristor-based chaotic systems. Then complete synchronization, proportional projective synchronization and function projective synchronization between two different memristor-based chaotic systems are realized, respectively. The synchronization characteristics are analyzed by error curve, synchronous sequence diagram, state variable fitting curves and phase diagrams, respectively. Numerical simulations are verified the correctness and feasibility of the proposed method.**

***Keywords-Synchronization for heterogeneous systems; Memristor-based chaotic systems; Projective synchronization***

## I. INTRODUCTION

In recent years, with the physical realization of the memristor and the extensive development of research on the memristor-based chaotic system, the research on synchronization control of memristor-based chaotic system has also aroused widespread concern of international and domestic academics [1-4]. In 1990, chaotic self-synchronization was first proposed by L.M. Pecora and T.L.Carrol of the United States Navy Laboratory. They used the drive-response method to synchronize two chaotic systems, which opened a new field of chaotic application. A new type of double-compound synchronization, based on combination-combination synchronization and compound synchronization of four chaotic systems, was investigated for six memristor-based Lorenz systems. Using Lyapunov stability theory and adaptive control, some sufficient conditions were attained to ensure the conclusions hold[5]. A novel complex Lorenz system with a flux-controlled memristor was investigated and its synchronization was realized [6]. Moreover, with the help of the memristor, the researchers constructed a new hyperchaotic system based on Chua circuit, and the synchronization of two chaotic systems with linear coupling was analyzed through the PC control and linear feedback method[7]. However, all of the above discussions focus on the synchronization between homogeneous memristor-based chaotic systems and there are few researches on synchronization of heterogeneous memristor-based chaotic systems. The synchronization of heterogeneous memristor-based chaotic systems is more complex than that of homogeneous memristor-based chaotic systems, so its anti-decipher ability is stronger, which has more advantageous to the application of secure communication, image encryption and many other applications of chaotic systems [8-11]. Therefore, it is very necessary to study the synchronization of heterogeneous memristor-based chaotic systems.

Based on Lyapunov stability principle, a nonlinear synchronization controller with exponential term is designed in this paper for two active magnetic controlled memristor-based chaotic systems with different structures. The designed nonlinear controller with different parameters is simple in structure and can realize complete synchronization, proportional projective synchronization and function projective synchronization between active magnetically controlled Lorenz-like memristor-based chaotic system and Chua-like memristor-based chaotic system. The realization of multiple projective synchronization of heterogeneous memristor-based chaotic systems lays a theoretical foundation for subsequent applications such as secure communication and image encryption.

## II. DESIGN OF SYNCHRONOUS CONTROLLER

Defining two n-dimensional dynamical systems with different structures. The drive system and controlled response system are described as, respectively,

$$\begin{cases} \dot{x}_1 = f_1\left(x_1, x_2, x_3, \cdots, x_n\right) \\ \dot{x}_2 = f_2\left(x_1, x_2, x_3, \cdots, x_n\right) \\ \dot{x}_3 = f_3\left(x_1, x_2, x_3, \cdots, x_n\right) \\ \quad\vdots \\ \dot{x}_n = f_n\left(x_1, x_2, x_3, \cdots, x_n\right) \end{cases} \tag{1}$$

and

$$\begin{cases} \dot{y}_1 = g_1\left(y_1, y_2, y_3, \cdots, y_n\right) + u_1 \\ \dot{y}_2 = g_2\left(y_1, y_2, y_3, \cdots, y_n\right) + u_2 \\ \dot{y}_3 = g_3\left(y_1, y_2, y_3, \cdots, y_n\right) + u_3 \\ \quad\vdots \\ \dot{y}_n = g_n\left(y_1, y_2, y_3, \cdots, y_n\right) + u_n \end{cases} \tag{2}$$

where $x_i, y_i, (i = 1, 2, 3 \cdots n)$ are the state variable of drive system and response system, respectively, $u_i (i = 1, 2, 3 \cdots n)$ is the synchronous controllers. To synchronize drive system with response system, the synchronization error between drive system and response system is given by

$$\lim_{t \to \infty} \left\| e_i \right\| = \lim_{t \to \infty} \left\| y_i - \alpha x_i \right\| = 0 \tag{3}$$

where the coefficient $\alpha \in \boldsymbol{R}^n \cap a \neq 0$ can be chosen different value. If, $\alpha$ is a setting proportional coefficient or a proportional function. If $\alpha = 1$, the synchronization for drive system and the response system belong to complete synchronization. When $\alpha \in \boldsymbol{R}^n \cap \alpha \neq 1 \cap a \neq 0$, the response system is synchronized with the drive system in different proportions. When $\alpha$ is a function of time, the controlled response system is function synchronized with the drive system.

In this way, the synchronization problem of two different systems can be converted into the control

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problem of the synchronous error system. The dynamic equation of the error system for drive system and response system is

$$\begin{aligned}
\dot{e}_1 &= g_1(y_1, y_2, y_3, ..., y_n) - \dot{\alpha}x_1 - \alpha f_1(x_1, x_2, x_3, ..., x_n) + u_1, \\
\dot{e}_2 &= g_2(y_1, y_2, y_3, ..., y_n) - \dot{\alpha}x_2 - \alpha f_2(x_1, x_2, x_3, ..., x_n) + u_2, \\
\dot{e}_3 &= g_3(y_1, y_2, y_3, ..., y_n) - \dot{\alpha}x_3 - \alpha f_3(x_1, x_2, x_3, ..., x_n) + u_3, \\
&\vdots \\
\dot{e}_n &= g_n(y_1, y_2, y_3, ..., y_n) - \dot{\alpha}x_n - \alpha f_n(x_1, x_2, x_3, ..., x_n) + u_n.
\end{aligned} \tag{4}$$

According to Lyapunov asymptotic stability theorem, a positive definite quadratic form function $V = (1/2)e^T e$ is designed as Lyapunov function. If there is $\dot{V} = \dot{e}^T e$, the system (4) is asymptotically stable on a large scale as long as $\dot{V}$ is negative semi-definite [12]. Therefore, the synchronous control problem of two systems can be indirectly transformed into the selection of control law and control parameters. In the following, two different memristor-based chaotic systems are taken as an example to design the synchronization controller and analyze its feasibility.

Let the Lorenz-like system in [13] be the drive system, which is an active magnetically controlled memristor-based chaotic system constructed on the classical Lorenz system. The governing equation is shown as

$$\begin{cases}
\dot{x}_1 = 36y_1z_1 - \alpha_1x_1 - z_1W(w_1) \\
\dot{y}_1 = \xi_1y_1 - \beta_1x_1z_1 \\
\dot{z}_1 = 8x_1y_1 - \gamma_1z_1 \\
\dot{w}_1 = x_1
\end{cases} \tag{5}$$

where $W(w_1) = a + 3bw_1^2$ is the mathematical model of three-order active magnetic controlled memristor, $w_1$ is the agnetic flux of the memristor, $a$ and $b$ are parameters that determine the characteristics of the memristor, $\alpha_1$, $\beta_1$, $\xi_1$ and $\gamma_1$ are the parameters that determine the motion state of the system, respectively.

Let the Chua-like system in [14] be the response system, which is an active magnetically controlled memristor-based chaotic system constructed on the classical Chua system. The corresponding equation is given by

$$\begin{cases}
\dot{x}_2 = \alpha_2(z_2 - W(w_2)x_2) + u_1 \\
\dot{y}_2 = \beta_2y_2 - z_2 + u_2 \\
\dot{z}_2 = y_2 - x_2 - \xi_2z_2 + u_3 \\
\dot{w}_2 = x_2 + u_4
\end{cases} \tag{6}$$

where $W(w_2) = a + 3bw_2^2$ is also the mathematical model of three-order active magnetic controlled memristor, $\alpha_2$, $\beta_2$ and $\xi_2$ is the parameters of the system,respectively. $u_1$, $u_2$, $u_3$ and $u_4$ are the synchronous controllers we designed, respectively.

The synchronization error is described as

$$\begin{cases}
e_1 = x_2 - \alpha x_1 \\
e_2 = y_2 - \alpha y_1 \\
e_3 = z_2 - \alpha z_1 \\
e_4 = w_2 - \alpha w_1
\end{cases} \tag{7}$$

Substituting (5) and (6) for (4), the following dynamic equations of error system are obtained

$$\begin{cases}
\dot{e}_1 = \alpha_2(z_2 - W(w_2)x_2) - \dot{\alpha}x_1 - \alpha(36y_1z_1 - \alpha_1x_1 - z_1W(w_1)) + u_1 \\
\dot{e}_2 = \beta_2y_2 - z_2 - \dot{\alpha}y_1 - \alpha(\xi_1y_1 - \beta_1x_1z_1) + u_2 \\
\dot{e}_3 = y_2 - x_2 - \xi_2z_2 - \dot{\alpha}z_1 - \alpha(8x_1y_1 - \gamma_1z_1) + u_3 \\
\dot{e}_4 = x_2 - \dot{\alpha}w_1 - \alpha x_1 + u_4
\end{cases} \tag{8}$$

Then select the following nonlinear feedback control functions

$$\begin{cases}
u_1 = (\dot{\alpha}/\alpha)x_2 + \alpha(36y_1z_1 - \alpha_1x_1 - z_1W(w_1)) \\
\quad - \alpha_2(z_2 - W(w_2)x_2) - k_1^{|sign(e_1)|}e_1 \\
u_2 = (\dot{\alpha}/\alpha)y_2 + \alpha(\xi_1y_1 - \beta_1x_1z_1) - \beta_2y_2 + z_2 - k_2^{|sign(e_2)|}e_2 \\
u_3 = (\dot{\alpha}/\alpha)z_2 + \alpha(8x_1y_1 - \gamma_1z_1) - y_2 + x_2 + \xi_2z_2 - k_3^{|sign(e_3)|}e_3 \\
u_4 = (\dot{\alpha}/\alpha)w_2 + \alpha x_1 - x_2 - k_4^{|sign(e_4)|}e_4
\end{cases} \tag{9}$$

where $k_i(i = 1,2,3,4)$ is the feedback control gain. For $\dot{\alpha} - k_i^{|sign(e_i)|} < 0$, the state variables of the drive system and the response system can be synchronized. Complete or proportional projective synchronization is achieved if $k_i > 0(i = 1,2,3,4)$ and $\alpha \in R$. The function projective synchronization between the two systems can be realized with $\dot{\alpha}/\alpha - k_i^{|sign(e_i)|} < 0(i = 1,2,3,4)$, when $\alpha$ is a function of time.

The feasibility of the synchronous controller is proved as follows. The error equations are obtained by combining (8) and (9)

$$\begin{cases}
\dot{e}_1 = (\dot{\alpha}/\alpha)e_1 - k_1^{|sign(e_1)|}e_1 \\
\dot{e}_2 = (\dot{\alpha}/\alpha)e_2 - k_2^{|sign(e_2)|}e_2 \\
\dot{e}_3 = (\dot{\alpha}/\alpha)e_3 - k_3^{|sign(e_3)|}e_3 \\
\dot{e}_4 = (\dot{\alpha}/\alpha)e_4 - k_4^{|sign(e_4)|}e_4
\end{cases} \tag{10}$$

Thus, the stability of the error system (8) can be directly analyzed to solve the synchronization problem of the drive and response systems.

Constructing Lyapunov function as $V = (1/2)e^T e$, the derivation of the Lyapunov function on time can be obtained as

$$\begin{aligned}
\dot{V} &= \dot{e}^T e = e_1\dot{e}_1 + e_2\dot{e}_2 + e_3\dot{e}_3 + e_4\dot{e}_4 \\
&= (\dot{\alpha}/\alpha - k_1^{|sign(e_1)|})e_1^2 + (\dot{\alpha}/\alpha - k_2^{|sign(e_2)|})e_2^2 \\
&\quad + (\dot{\alpha}/\alpha - k_3^{|sign(e_3)|})e_3^2 + (\dot{\alpha}/\alpha - k_4^{|sign(e_4)|})e_4^2
\end{aligned} \tag{11}$$

As long as $\dot{\alpha}/\alpha - k_i^{|sign(e_i)|} < 0(i = 1,2,3,4)$, $\dot{V}$ must be a negative semi-definite, the projective synchronization of drive system and response system is realized. Drive system and response system can achieve complete or proportional projective synchronization when $\alpha \in R$ and $k_i > 0$ $(i = 1,2,3,4)$. When $\alpha$ is a function about time, there are many combinations about the choice of $\alpha$ and $k_i(i = 1,2,3,4)$, the function projective synchronization between the two systems can be realized with $\dot{\alpha}/\alpha - k_i^{|sign(e_i)|} < 0(i = 1,2,3,4)$.

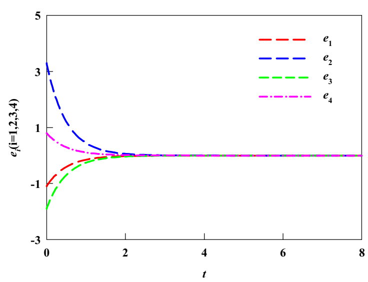
## III. Projective Synchronization for Heterogeneous Memristor-based Chaotic Systems

The system parameters in (5) are set as $a = -0.5$, $b = 0.8$, $\alpha_1 = 15$, $\beta_1 = 8$, $\xi_1 = 1.68$, and $\gamma_1 = 15.15$, and the system parameters in (6) are given by $a = -0.4$, $b = 0.8$,

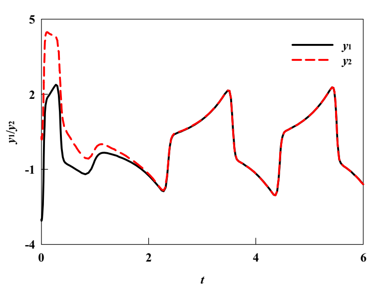
$\alpha_2 = 4$, $\beta_2 = 0.7$, $\xi_2 = 0.1$. Based on previous analysis, the multiple projective synchronization can be realized by changing the value of the feedback control gain $k_i\,(i=1,2,3,4)$ and $\alpha$.

### A. Complete Synchronization

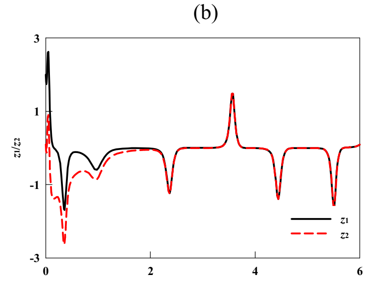
For $\alpha = 1$, make sure $k_i > 0\,(i=1,2,3,4)$ and the complete synchronization of the drive system and the response system can be realized. Let $k_1 = k_2 = k_3 = k_4 = 2$, the initial conditions of the drive system and the response system are presented as $(x_{1o1}, y_{1o1}, z_{1o1}, w_{1o1}, x_{2o1}, y_{2o1}, z_{2o1}, w_{2o1})$ = $(1,3,2,1,0.1,0.3,0.1,0.2)$ respectively. Then complete projective synchronization of the two systems is achieved as shown in Fig.1. The synchronization error curves of complete synchronization are tend to zero for $t \geq 2s$ or so in Fig.1 (a). It can be seen that the heterogeneous memristor-based chaotic systems reach gradually complete synchronization. The time-histories of drive system and response system are also shown in Fig.1 (b), (c), (d), respectively.



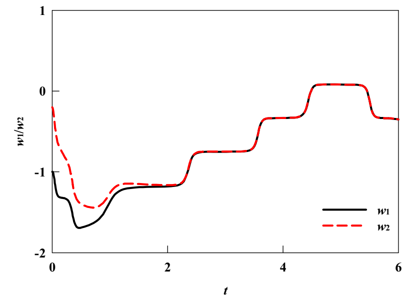
(a)



(b)



(c)



(d)

**Fig. 1** Complete synchronization simulation diagram:(a) the error curves for synchronization, (b)-(d) time-histories of the state variables $y_1$ and $y_2, z_1$ and $z_2, w_1$ and $w_2$.

### B. Proportional Projective Synchronization

When $\alpha \in \boldsymbol{R}^n \cap \alpha \neq 1 \cap \alpha \neq 0$, just make sure $k_i > 0\,(i=1,2,3,4)$ and the proportional projective synchronization will be realized. When $\alpha$=2, $k_1 = 1, k_2 = 2, k_3 = 3, k_4 = 4$ are given, and the initial conditions of drive system in (5) and response system in (6) are also shown as $(1,3,2,1,0.1,0.3,0.1,0.2)$, proportional projective synchronization for heterogeneous memristor-based chaotic systems is achieved as depicted in Fig.2. Taking $\alpha = 2$ means that the value of each state variable in the response system is doubled by the corresponding variable of the drive system. The black region in Fig. 2 is the attractor produced by the drive system while the red region is the attractor produced by the response system. The projective synchronization ratio between the response system and the drive system can be clearly observed from the diagram, which meets the design requirements of the synchronization controller.

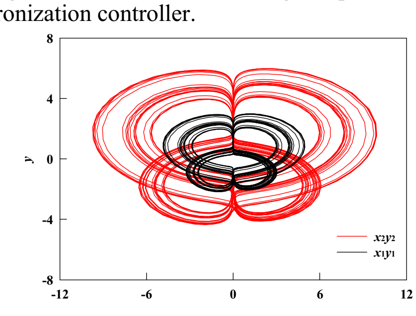



**Fig.2** The phase plane for proportional projective synchronization

### C. Function Projective Synchronization

When $\alpha$ is a function of time, and $\dot{\alpha}/\alpha - k_i^{|sign(e_i)|} < 0$, $(i=1,2,3,4)$ is satisfied at the same time, the functional synchronized will be obtained. Let $\alpha = -2 + 0.5\sin t$ and $k_1 = k_2 = k_3 = k_4 = 2$, then we can figure out $-k_i^{|sign(e_i)|}(i=1,2,3,4) \in \{-1,-2\}$, $\dot{\alpha} = 0.5\cos t \in [-0.5, 0.5]$, then $\dot{\alpha}/\alpha - k_i^{|sign(e_i)|} < 0\,(i=1,2,3,4)$ is satisfied. To contrast with proportional projective synchronization, the initial values of drive system in (5) and response system in (6) are also set as (-1,1,2,-1,0.1,0.2,-0.3,-0.1). Then the functional projective synchronization for heterogeneous memristor-based chaotic systems is achieved and plotted in Fig. 3.

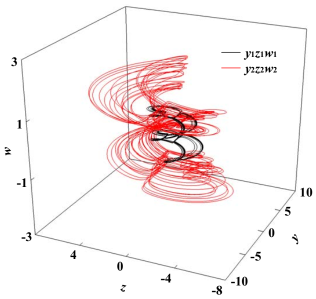



**Fig.3** Phase plane of functional projective synchronization

## IV. CONCLUSION

In this paper, the projective synchronization between the Lorenz-like and the Chua-like memristor-based chaotic systems was realized. The nonlinear synchronous controllers with exponential terms were designed according to the equations of the active magnetic control memristor-based chaotic systems to realize the projective synchronization. Through the analysis of the error curves, the synchronous sequence diagram, the time-histories of state variables and the phase diagrams, the different synchronizations of two systems with different structures were obtained. The synchronization controller designed in this paper has good synchronization performance, and the simulation results were in good agreement with the theoretical analysis. Memristor-based chaotic system has special and rich dynamic phenomena, and the synchronization type of heterogeneous chaotic systems is more complex than homogeneous chaotic systems. Therefore, the projective synchronization of active magnetically controlled heterogeneous memristor-based system in this paper has great application prospect in chaotic secure communication, image encryption and so on.

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