

## An Analytical Model of Distributed Energy Storage Systems in Power Distribution Networks

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**Abstract**— Distributed Energy storage system (ESS) has a significant impact on the flexibility of medium/low voltage power distribution network to address the challenges. This paper explicitly quantifies the potential benefit of optimal coordinated multiple ESSs to support the secure power supply of power distribution networks with distributed generations (DGs) by providing capacity services. The proposed model was assessed through case studies: different demand conditions, different sizing of ESSs and reactive power provisioning. The numerical result demonstrates various benefit of coordination of ESSs in supporting flexible and secure operation of power distribution networks, e.g. peak demand reduction, reactive power support.

**Keywords**—Distributed energy storage, power distribution networks;

### Parameters

$E_0$	Initial energy level of ESS	[MWh]
$\eta_i$	Round-trip efficiency of ESS $i$	
$S_{ij}^{Max}$	Apparent power limit of distribution line from bus $i$ to $j$	[MVA]
$P_{i,t}^d$	Active power demand at node $i$ in hour $t$	[MW]
$Q_{i,t}^d$	Reactive power demand at node $i$ at hour $t$	[MVA]
$D_i^{Max}$	Maximum discharging power of ESS $i$	[MW]
$S_i^{Max}$	Maximum apparent power of ESS $i$	[MVA]
$C_i^{Max}$	Maximum charging power of ESS $i$	[MW]
$E_i^{Max}$	Energy capacity of ESS $i$	[MWh]
$\pi^D$	Annuitized distribution network reinforcement cost	[£/MWh]
$\pi_t$	Energy price in hour $t$	[£/MWh]

### Variables

$P_t^{ij}$	Active power flow from bus $i$ to $j$ in hour $t$	[MW]
$Q_t^{ij}$	Reactive power flow from bus $i$ to $j$ in hour $t$	[MVA]
$P^{GI}$	Active power imported from the grid in hour $t$	[MW]
$P_{i,t}^{DG}$	Active power supplied by DG $i$ in hour $t$	[MW]
$Q_{i,t}^{DG}$	Reactive power supplied by DG $i$ in hour $t$	[MVA]
$P_{i,t}^{ND}$	Active power net demand at node $i$ in hour $t$	[MW]
$Q_{i,t}^{ND}$	Reactive power net demand at node $i$ at hour $t$	[MVA]
$E_{i,t}$	Energy level of ESS $i$ in hour $t$	[MWh]
$C_{i,t}$	Active charging power of ESS $i$ in hour $t$	[MW]

$Q_{i,t}^C$	Reactive charging power of storage $i$ in hour $t$	[MVA]
$D_{i,t}$	Active discharging power of storage $i$ in hour $t$	[MW]
$Q_{i,t}^D$	Reactive discharging power from storage $i$ in hour $t$	[MVA]
$S^{Peak}$	Reduction in overall peak demand	[MVA]

### I. INTRODUCTION

In recent years, the power distribution networks have been faced increasing challenges as growing penetration of renewable energy sources (RES) in the form of small-scale renewable distributed generations (DGs) [1] (e.g., wind turbines, solar energy [2] and Combined Heat and Power (CHP) [3]). This is mainly driven by the advances in smart grid technologies and the necessary decarbonisation of the electricity sector. However, such changes to the current passive distribution networks brought direct challenges, e.g. voltage raise effect, increased fault level, protection degradation, and altered transient stability. This requires the power distribution network requires having sufficient flexibility to ensure technically and economically effective operation of its infrastructure [4]. To address the emerging technical challenges, smart grid control and management solutions, such as coordinated voltage control (CVC), soft-open point (SOP), demand-side response (DSR) [5], active network management [6] and energy storage systems (ESSs), have been adopted as viable options.

ESS has the potential to provide multiple services to the power system, which can benefit several sectors (generation, transmission network operators, distribution network operators, and the demand side) across the system and facilitate the transformation of the existing power system into a future low-carbon one [7]-[10]. Hence, to coordinate simultaneous operations and aggregate the maximum values of the ESSs are of great importance. EES can be adopted to improve power system security and stability by addressing the uncertainty associated with RES [11]. A Mixed Integer Linear Programming (MILP) model was presented in [12] to maximize the net profit of the ESS with the provision of multiple services, such as energy arbitrage, balancing services and network congestion management through both active and reactive power control. In addition, the payoff of ESS investment for multi-agent players in the energy market was considered in [13].

However, the aforementioned solutions have provided limited insights for the potential economic and technical values that ESSs can present to the distribution network with DGs. To this end, this work evaluates and quantifies the benefits of coordinated distributed ESS for power distribution network operation from the DNO's

perspective are evaluated and quantified. The main technical contributions made in this work are as follows: As DNOs can deploy ESS to deferral network capacity upgrade, reduce the peak demand and manage reactive power flow. Moreover, an analytical model for distribution network operator with distributed ESS to optimise the value of ESS is identified and examined through case studies, which can facilitate the transformation of the current power system into a future low-carbon system. The rest of the paper is organized as follows: the problem formulation is presented in Section II; the solution is evaluated through a set of case studies and the numerical results are provided in Section III; and finally the conclusive remarks are given in Section IV.

## II. PROBLEM FORMULATION AND PROPOSED MODEL

In this work, a simplified 3-bus interconnected power network model is used to study the proposed solution, as shown in Fig. 1. Such simplified network structure can well represent the operational characteristics of network operation and the interrelations between power demand and ESS operation at individual buses.

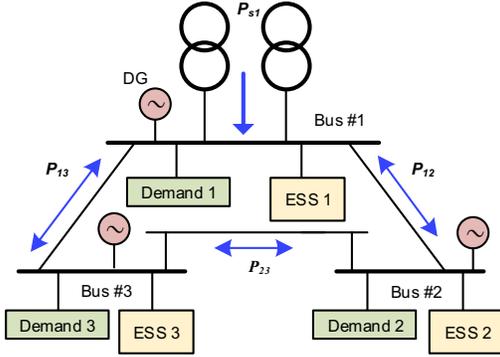


Figure 1. The adopted 3-bus network model (arrows: power flow).

In this presented system, DG and ESS were installed near the local demand at each bus. The net demand at one bus is expressed as its local demand minus the power supplied by local DG. Moreover, the grid supplying demand is defined as the power imported from the grid except for DG and ESS supply at each bus. In this sense, the analytical model was designed to minimise the cost of grid power import considering the economic benefits provided by ESSs. As a result, the objective function of the proposed solution is to minimize the cost of the imported active power  $P^{GI}$  by power distribution network from the grid regarding the energy arbitrage and annuitized network capacity reinforcement cost avoided by peak system demand shaving. The revenue and cost savings can be obtained through the power arbitrage service and DNO service provided by ESSs, respectively.

The DNO service is attained by the peak system demand reduction, which leads to the deferment or even avoiding of the power distribution network reinforcement and capacity expansion. The revenue gained by energy arbitrage during a given period is determined by the product of the energy sold  $D_{i,t}$  or bought  $C_{i,t}$  to or from the energy market and its corresponding the energy price  $\pi_t$ . The cost savings are determined by the amount of

reduction in annual peak system demand  $S^{Peak}$  multiplied by the annuitized distribution network reinforcement cost  $\pi^D$ . Thus, the objective function is given as below:

$$\min \left\{ \sum_{t=1}^T \left( P_t^{GI} + \sum_{i=1}^3 (C_{i,t} - D_{i,t}) \right) \times \pi_t - S^{Peak} \times \pi^D \right\} \quad (1)$$

The local balance for both reactive and active power between demand and supply is described through (4)-(7). The power flows into one node is expressed as the power flow from the other nodes and generation sources (i.e. grid generation, DGs and ESS) minus the power utilized by the local demand. Furthermore, the output of an ESS is defined as its discharging power reduced by the charging power in hour  $t$  through (8)-(10), which holds for both active and reactive power. The power distribution network line capacity limits are presented in (10).

$$P_{i,t}^{ND} = P_{i,t}^d - P_{i,t}^{DG} \quad (2)$$

$$Q_{i,t}^{ND} = Q_{i,t}^d - Q_{i,t}^{DG} \quad (3)$$

$$P_t^{S1} + P_{1,t}^{output} = P_{1,t}^{ND} + P_t^{12} + P_t^{13} \quad (4)$$

$$Q_t^{S1} + Q_{1,t}^{output} = Q_{1,t}^{ND} + Q_t^{12} + Q_t^{13} \quad (5)$$

$$\sum_{i=1}^3 P_t^{ij} + P_{j,t}^{output} = P_{j,t}^{ND} + \sum_{i=1}^3 P_t^{ji} \quad (6)$$

$$\sum_{i=1}^3 Q_t^{ij} + Q_{j,t}^{output} = Q_{j,t}^{ND} + \sum_{i=1}^3 Q_t^{ji} \quad (7)$$

$$P_{i,t}^{output} = D_{i,t} - C_{i,t} \quad (8)$$

$$Q_{i,t}^{output} = Q_{i,t}^D - Q_{i,t}^C \quad (9)$$

$$(Q_t^{ij})^2 + (P_t^{ij})^2 \leq (S_{ij}^{Max})^2 \quad (10)$$

The ESS can be considered as a prosumer of energy, which provides flexibility to the network by acting either as a generator through discharging power or load through charging power. In hour  $t$ , the energy level of an ESS is expressed in (11) as the its previous energy level plus the charging and discharging actions at time  $t$ , which are affected by the efficiency.

$$E_{i,t} = E_{i,t-1} + \eta_i \times C_{i,t} - D_{i,t} \times \frac{1}{\eta_i} \quad (11)$$

The coordinated operation of active and reactive power for each ESS is bound by its maximum charging and discharging capacity limits, which are demonstrated through (12)-(15). Moreover, the energy level of each ESS is subject to its maximum energy capacity express in (16)

$$C_{i,t} \leq C_i^{Max} \quad (12)$$

$$(Q_{i,t}^C)^2 + (C_{i,t})^2 \leq (S_i^{Max})^2 \quad (13)$$

$$D_{i,t} \leq D_i^{Max} \quad (14)$$

$$(Q_{i,t}^D)^2 + (D_{i,t})^2 \leq (S_i^{Max})^2 \quad (15)$$

$$E_{i,t} \leq E_i^{Max} \quad (16)$$

Due to incapability of the optimization solver (Fico Express) to process nonlinear constraints, such as (13) and (15), a linearization method is adopted to approximate the search space in a convex linearized region, as shown in Fig. 2. The green area shown demonstrates the nonlinear relationship between the active and reactive power, which is confined by the apparent power limit as expressed in (17). The linearization method is achieved by implementing the lines tangent to the circle to enclose an area shown as the blue line area in order to approximate the green area. Note that to get a comparatively more accurate approximation more tangent lines are preferred;

meanwhile the trade-off of a high accuracy may sacrifice the convergent speed of the optimization.

$$Q^2 + P^2 \leq S_{max}^2 \quad (17)$$

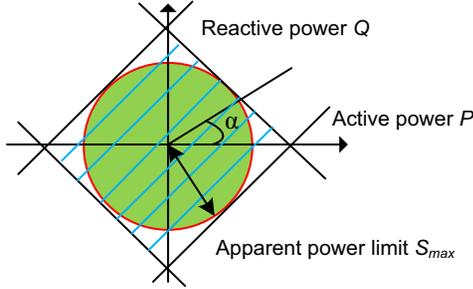


Figure 2. Power linearization diagram at four quadrants.

The tangent lines at each quadrant presented in Fig.2 are expressed through (18-21), with its parameters defined through (22-23).

$$Q \leq -P \times k_i + b_i \times S_{max}, 0^\circ \leq \alpha < 90^\circ \quad (18)$$

$$Q \leq P \times k_i + b_i \times S_{max}, 90^\circ \leq \alpha < 180^\circ \quad (19)$$

$$Q \geq -P \times k_i - b_i \times S_{max}, 180^\circ \leq \alpha < 270^\circ \quad (20)$$

$$Q \geq P \times k_i - b_i \times S_{max}, 270^\circ \leq \alpha < 360^\circ \quad (21)$$

$$k_i = -\frac{1}{\tan \alpha}, 90^\circ \leq \alpha < 180^\circ \quad (22)$$

$$b_i = \sin \alpha + \cos \alpha \times \frac{1}{\tan \alpha}, 90^\circ \leq \alpha < 180^\circ \quad (23)$$

The proposed linearization method can be implemented through the following steps:

Step (1): Initialization of the parameters, set an appropriate number of tangent lines at each quadrant as  $N = 20, i = 0$ ;

Step (2): If  $i \leq N$  then go to step 3, else go to step 4;

Step (3): Set  $\alpha = \frac{90^\circ}{N} \times i - 2.5^\circ$ ,  $k_i = -\frac{1}{\tan \alpha}$ ,  $b_i =$

$\sin \alpha - \cos \alpha \times k_i$ .  $k_i$  is the slope of the black tangent line to the circle shown in Fig.2 and  $b_i$  is the y-intercept;

Step (4): The feasible region is the area enclosed by the lines obtained regarding previous steps.

### III. CASE STUDY

In this case study, considering the presence of DGs at individual buses with the penetration of 30%, the power demand can be supplied by both connected DGs and power utility. The profile of the power demand that needs to be further supplied by the power grid or ESS (i.e. net demand) is assumed following the pattern of a real UK 33/11 kV primary substation measurements with the demand power factor of 0.96. The real UK electricity price of year 2014 is adopted with an hourly resolution [14]. The distribution network annuitized cost was extracted from [15]. In addition, the line capacity of the distribution network is presented Table 1.

TABLE I. DISTRIBUTION NETWORK LINE CAPACITY SETTINGS

From node	To node	Secured capacity (MVA)
1	2	50
1	3	35
2	3	35
Grid	1	150

The adopted parameters for the proposed analytical model of coordinated ESS in the following case studies

are presented in Table 2. Note that the ESS operation is not only subject to its apparent power capacity limit (in MVA), but also to its energy capacity (in MWh).

It is known that the electric power consumption is often sensitive to weather conditions -such as temperature and humidity, which can also affect the local electricity price as it is correlated to the local electricity demand. However, the patterns of changing daily demand conditions are relatively consistent across yearly seasons (i.e. winter, spring, summer and autumn). The benefits of coordinated ESSs for the operation of power distribution networks are firstly evaluated in typical days for different seasons. In such context, energy arbitrage opportunities are seized by taking the advantage of price differences between the early morning off-peak price and the evening peak price, which indicates that the energy arbitrage service provided by ESS will be affected by the seasonal energy price variations.

TABLE II. ESS PARAMETER SETTING

Node	Power limit (MVA)	Active power limit (MW)	Energy capacity (MWh)	Round-trip efficiency (%)
1	2	2	4	0.85
2	5	4	5	0.85
3	5	3	10	0.85

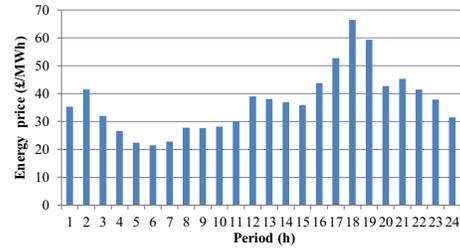


Figure 3. Energy price of a spiky demand day in winter.

Fig.3 and Fig.4 (a) demonstrate that ESS is buying energy at periods with lower energy prices to charge up to its maximum capacity, and selling this energy back to the electricity market by discharging when the energy is more valuable. Fig.4 (a) indicates that ESS charges power from 4:00 h to 6:00 h, which is consistent with the off-peak energy price period observed in Fig.3 as from 4:00 h to 6:00 h in the early morning. In turn, the storage discharges energy from 17:00 h to 19:00 h, which is also correlated to the peak energy price period. Additionally, it is noticeable that the ESS charges more energy compared to the amount of that discharged, the possible explanation is the energy loss due to round-trip efficiency as shown in (11). Moreover, arbitrage actions can be affected by this energy loss, since the cost of energy loss needs to be offset by the revenue obtained from the energy arbitrage.

However, the peak local active demand at 19:00 h is not reduced by the discharging action of ESS shown in Fig.4 (a). This is due to that the local peak demand occurs at 18:00 h, which is right before the peak system demand that is consistent with the energy price peak shown in Fig.3. The time difference leads to a conflicting relationship between DNO service and energy arbitrage since the stored energy level is inadequate to offer two services consecutively in this case. However, the reactive

power demand is reduced by the continuously reactive power output support from storage, with approximately 50% peak demand reduction. In this case, reactive power output from storage can be used to offset the increase in active net demand and remain DNO service provided, e.g. between periods 19:00 h to 21:00 h ESS is discharging reactive power to offset increase in active net demand.

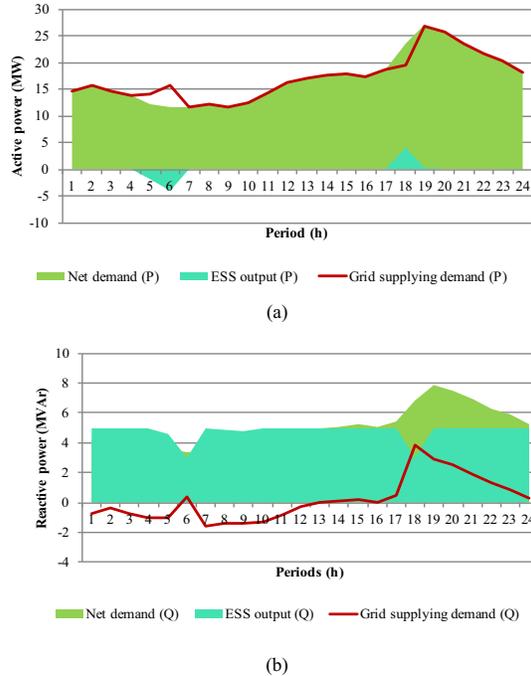


Figure 4. (a) Active and (b) reactive power of a spiky demand day in winter.

#### IV. CONCLUSIONS AND REMARKS

This paper investigated the benefit of distributed ESSs for supporting the flexible and secure operation of power distribution networks. A collection of insights are obtained and motivate the future work as follows:

Firstly, the benefit quantification of coordinated ESSs in power distribution networks is investigated using a simplified 3-bus network model. The analytical approach can be applied and further validated using large-scale network scenarios. In addition, this work has clearly demonstrated the significant value of coordinated ESS facility in terms of active and reactive power support in DNO service provisioning and energy arbitrage. As a result, the investment cost due to power distribution network infrastructure reinforcement and capacity expansion can be effectively deferred or even avoided in coping with the variability due to the increasing power demand and penetration of DGs. As an additional benefit, the DNOs can further obtain revenues through the arbitrage actions from the power market. However, an appropriate market mechanism that can effectively reward the value of coordinated ESS operation still demands further research exploration. Finally, the inappropriate

management or even failure of ESS operation can directly affect the performance of power supply security. To ensure stable and secure operation of the power distribution networks with coordinated ESSs, the mechanisms for guaranteeing reliable operation of coordinated ESS facilities need to be further studied.

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