A Data Assimilation Enabled Model for Coupling Dual Porosity Flow with Free Flow

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***Abstract*— Coupling of dual porosity flow and free flow arises in many important applications, e.g., groundwater system and industrial filtrations. Existing Stokes-Darcy types of models cannot accurately describe this type of coupled problem since they only consider single porosity media. With the support of lab experiment data we are developing a new coupled multi-physics, multiscale model and an efficient numerical method to solve it. Furthermore, both the lab and field data provide the possibility to improve the accuracy of the model prediction through data assimilation.**

Keywords-component: fluid flow; reservoir modeling; data assimilation; high performance computing

I. Introduction

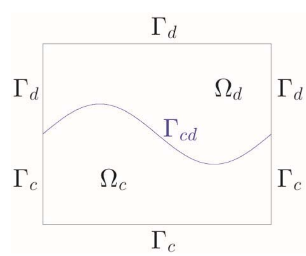
In many real world problems and industrial settings, the free flow of a liquid and the confined flow in a dual porosity media are often coupled together and significantly affected by each other. However, the existing Stokes-Darcy types of models cannot accurately describe this type of coupled problem since they only consider single porosity media. Therefore, with the support of lab experiment data, we follow the general framework of Stokes-Darcy model and dual-porosity model to develop a new coupled multi-physics multi-scale model and the corresponding numerical methods for accurately describing the coupling of the flow in dual-porosity media and the free flow. The resulting coupled dual porosity Navier-Stokes model has higher fidelity than the Darcy, dual porosity, Navier-Stokes, or Stokes-Darcy equations on their own. Furthermore, the field data provides the possibility to improve and demonstrate the accuracy of the model prediction through data assimilation.

The coupling of porous media flow and free flow arises in many important applications, e.g., (a) hydrology problems, carbon sequestration, geothermal systems, and petroleum extraction in fractured reservoirs/aquifers or around horizontal wellbores, (b) coupled surface and subsurface flow, (c) biochemical transport and field-flow fractionation, (d) blood motion in lungs, solid tumors and vessels, (e) the mushy zone in alloy solidification, and (6) topology optimization of fluid flows.

A traditional model for this type of coupling problems is the Stokes-Darcy model, which describes the free flow by the Stokes equation and the porous media flow by the Darcy's law, and then couples these two equations through three interface conditions. While dual-porosity models have been widely used to describe naturally fractured porous media for different problems in hydrology, carbon sequestration, geothermal system, and petroleum extraction [1-7], this model itself does not consider the free flow in large conduits nor do existing Stokes-Darcy models consider a dual-porosity model when they couple the porous media flow with the free flow. Hence a new dual-porosity-Navier-Stokes model is needed for a more accurate description of the coupled dual-porosity flow and free flow.

II. Models

First consider a simple example of a dual porosity subdomain Ω_d and a conduit subdomain Ω_c, where $\Omega = \Omega_d \cup \Omega_c$, $\Omega_d \cap \Omega_c = \emptyset$, and the interface between the two subdomains is Γ_{cd}, which is represented by the following:



A traditional dual porosity model is given in Ω_d with matrix and micro fracture equations [8]:

$$\phi_m C_{mt} \frac{\partial p_m}{\partial t} - \nabla \cdot \left(\frac{k_m}{\mu} \nabla p_m \right) = -Q, \tag{1}$$

$$\phi_f C_{ft} \frac{\partial p_f}{\partial t} - \nabla \cdot \left(\frac{k_f}{\mu} \nabla p_f \right) = Q + q_p, \tag{2}$$

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where μ is the dynamic viscosity, q_p is the source/sink term, $Q = \frac{\sigma k_m}{\mu}(p_m - p_f)$ measures the mass exchange between matrix and micro fractures, σ is a shape factor characterizing the morphology dimension of the micro fractures, k_m/k_f represents the intrinsic permeability in the matrix or fracture, p_m/p_f represents the pressure in the matrix or fracture, ϕ_m/ϕ_f represents the porosity in the matrix or fracture, and C_{mt}/C_{ft} represents the total compressibility in the matrix or fracture.

In the conduit Ω_c, the flow is governed by a Navier-Stokes equation,

$$\frac{\partial \boldsymbol{u_c}}{\partial t} + (\boldsymbol{u_c} \cdot \nabla)\boldsymbol{u_c} - \nabla \cdot \mathbb{T}(\boldsymbol{u_c}, p) = \boldsymbol{f}, \nabla \cdot \boldsymbol{u_c} = 0, \tag{3}$$

where $\boldsymbol{u_c}$ is the velocity, p is the kinematic pressure, $\mathbb{T}(\boldsymbol{u_c}, p) = 2\upsilon\mathbb{D}(\boldsymbol{u_c}) - p\mathbb{I}$ is the stress tensor, $\mathbb{D}(\boldsymbol{u_c}) = \frac{1}{2}(\nabla \boldsymbol{u}_c + \nabla \boldsymbol{u_c^T})$ is the deformation tensor, $\mathbb{I}$ is the identity matrix, υ is the kinematic viscosity of the fluid, and $\boldsymbol{f}$ is a general body forcing term.

To combine the two separate models into a coupled system, we need four interface conditions on the interface Γ_{cd} based on the following three fundamental properties of dual-porosity media.

- The matrix permeability in a dual-porosity media is critically low compared with the micro-fracture permeability. For example, in a shale or tight reservoir, the matrix permeability is usually 10^5 to 10^7 times smaller than the micro-fracture permeability.
- The matrix porosity is usually much larger than the micro-fracture porosity. For example, in a shale or tight reservoir, the matrix porosity is usually 10^2 to 10^3 times larger than the micro-fracture porosity.
- The shape factor σ, which ranges from 0 to 1, can be determined according to the morphology and dimension of the micro fractures by using different types of formulas.

See [9-13].

In the dual-porosity media with the above properties, the matrix system serves as the main storage space and the micro fracture system serves as the preferential fluid movement channel. Due to the critically low permeability in the matrix and the much faster flow in the micro fractures, the dual porosity model neglects the flows between the matrix and the conduits/macro fractures. That is, the dual porosity model assumes that the fluid drains from the matrix block into the adjacent micro fractures and then into the conduits/macro fractures. Since the matrix is assumed to only feed the micro fractures, the conduits/macro fractures do not directly communicate with the matrix, but only communicate with the micro fractures [14-18].

Following the idea in [19], the four interface conditions are given below.

1. A no exchange condition between the matrix and the conduits/macro-fractures:

$$-\frac{k_m}{\mu}\nabla p_m \cdot (-\boldsymbol{n_{cd}}) = 0, \tag{4}$$

where $\boldsymbol{n_{cd}}$ is the unit normal vector on the interface edges pointing from Ω_c to Ω_d.

2. Conservation of mass:

$$\boldsymbol{u}_c \cdot \boldsymbol{n_{cd}} = -\frac{k_f}{\mu}\nabla p_f \cdot \boldsymbol{n_{cd}}. \tag{5}$$

3. Balance of forces:

$$\boldsymbol{n_{cd}}\mathbb{T}(\boldsymbol{u_c}, p)\boldsymbol{n_{cd}^T} = \frac{p_f}{\rho}. \tag{6}$$

4. Beavers-Joseph condition [20]:

$$-\mathbb{P}_\tau(\mathbb{T}(\boldsymbol{u_c}, p)\boldsymbol{n_{cd}}) = \frac{\theta \nu \sqrt{D}}{\sqrt{trace(\boldsymbol{\Pi})}}\mathbb{P}_\tau\left(\boldsymbol{u_c} + \frac{k_f}{\mu}\nabla p_f\right), \tag{7}$$

where $\mathbb{P}_\tau$ is the projection onto the local tangent plane on Γ_{cd}, θ is the Beavers-Joseph coefficient, $\boldsymbol{\Pi} = k_f\mathbb{I}$ is the intrinsic permeability of fracture medium, and D is the number of spatial dimensions.

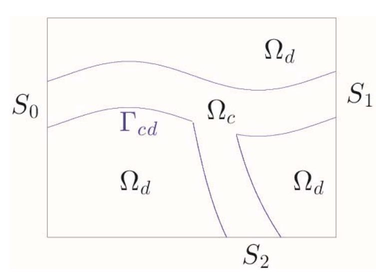
The interface conditions play a key role and usually cause the major difficulty for interface problems.

Finally, we need boundary and initial conditions in order to have a well posed system. Either Dirichlet or Neumann conditions are needed for the variables p_f and p_m on Γ_d, $\boldsymbol{u}_c$ on Γ_c, $p_m(\boldsymbol{x}, 0)$, $p_f(\boldsymbol{x}, 0)$, and $\boldsymbol{u}_c(\boldsymbol{x}, 0)$.

Usually it is extremely difficult and expensive to measure the fluid flow velocity to obtain the velocity data [21]. It is easier to obtain the flow rate data for the following defective boundary conditions on portions of the domain boundary S_i, $i = 0,1,\cdots,m$ [22]:

$$\int_{S_i} \boldsymbol{u_c} \cdot \boldsymbol{n_{cd}} ds = Q_i, \qquad i = 0,\cdots,m. \tag{8}$$

Going back to the example at the beginning of this section, for m=2, we could have



The solutions of a dual porosity Navier-Stokes model with these conditions are not unique and utilizing Lagrange multipliers for the defective boundary conditions leads to a saddle point problem. This boundary condition is difficult to use in the solvers we are developing.

III. TRANSMISSION CONDITIONS FOR A MULTI PHYSICS DOMAIN DECOMPOSITION

A noniterative multiphysics domain decomposition method (MPDDM) has been developed [23-26] for solving the interface problem and is used at each time step of the data assimilation process described in Section IV. We avoid using an explicit scheme while using the information in the preceding time steps efficiently. At each time step, our method only needs a single multi-phase Navier-Stokes solve and a single multi-phase Darcy solve in parallel, which is optimal.

We reorganize all the components in the physical interface conditions into Robin type boundary conditions on the interface with relaxation parameters specifically designed for the different scales of Darcy and Stokes flows in order to decompose different physical subdomains according to relevant physics in the multiphysics setting. Based on the normal interface conditions (4)-(7), the mixed transmission conditions on the interface are

$$-\frac{k_m}{\mu}\nabla p_m \cdot (-\boldsymbol{n_{cd}}) = 0, \tag{9}$$

$$\gamma_d \frac{k_f}{\mu}\nabla p_f \cdot \boldsymbol{n}_{cd} + \frac{p_f}{\rho} = \xi_d, \tag{10}$$

$$\boldsymbol{n}_{cd}^T \mathbb{T}(\boldsymbol{u}_c, p)\boldsymbol{n}_{cd} + \gamma_c \boldsymbol{u}_c \cdot \boldsymbol{n}_{cd} = \xi_c, \tag{11}$$

$$-\mathbb{P}(\mathbb{T}(\boldsymbol{u}_c, p)\boldsymbol{n}_{cd}) - \frac{\theta \upsilon \sqrt{D}}{\sqrt{trace(\boldsymbol{\Pi})}}\mathbb{P}_\tau \boldsymbol{u}_c = \xi_{CT} \text{ on } \Gamma_{cd}, \tag{12}$$

where ξ_d, ξ_c, and ξ_{CT} denote three auxiliary functions on Γ. Each component on the left sides of these mixed conditions directly comes from the three interface conditions. These four conditions enable us to decompose the original coupled system into two subproblems, the dual-porosity equation and the Navier-Stokes equation. Therefore, the auxiliary functions ξ_d, ξ_c, and ξ_{CT} play a key role in the decomposition and the construction of the non-iterative algorithm, especially ξ_{CT} since it is responsible for the additional term from the Beavers-Joseph condition. One advantage of these natural Robin type conditions is that it is not complicated to obtain the equivalence between the decoupled system and the original one. These equivalence conditions provide convenient tools to directly predict ξ_d, ξ_c, and ξ_{CT} on the interface at each time step based on the results from the previous time steps [27,28]. Then we can solve the decoupled dual-porosity and Navier-Stokes equations independently at each time step based on the predicted ξ_d, ξ_c, and ξ_{CT}.

IV. DATA ASSIMILATION

In order to fully make use of the field data to improve the prediction of the proposed model, we plan to develop a variational data assimilation method. For the variational data assimilation method, we can consider the following general evolution problem to illustrate the basic idea [29]:

$$\frac{\partial \varphi}{\partial t} = \mathcal{F}(\varphi), t \in (0, T), \varphi|_{t=0} = u, \tag{13}$$

where $\varphi = \varphi(t)$ is the unknown function belonging for any t to a Hilbert space $H, u \in H$, and $\mathcal{F}: H \to H$ is a nonlinear operator determined by the model. Let $Y = L_2([0,T], H)$ and $\|\cdot\|_Y = (\cdot,\cdot)_Y^{1/2}$. Define the cost functional by

$$J(u) = \frac{\alpha}{2}\|u\|_H^2 + \int_0^T \|C\varphi - \hat{\varphi}\|_H^2 dt, \tag{14}$$

where $\alpha \in \mathbb{R}^+$, $\hat{\varphi} \in Y_{obs} \subset Y$ is the observation, and $C: Y \to Y_{obs}$ is a linear operator. Then the variational data assimilation problem is to find the optimal control u that minimizes the cost functional $J(u) = \inf_{v \in H} J(v)$ subject to equation (13). Following [29], the necessary optimality condition reduces the problem (13) to the system, where * represents the adjoint and $'$ represents the Frechet derivative:

$$\begin{cases} \frac{\partial \varphi}{\partial t} = \mathcal{F}(\varphi), t \in (0,T), \\ -\frac{\partial \varphi^*}{\partial t} - \left(\mathcal{F}'(\varphi)\right)^* \varphi^* = -C^*(C\varphi - \hat{\varphi}), t \in (0,T), \\ \varphi|_{t=0} = u, \varphi^*|_{t=T} = 0, \alpha u - \varphi^*|_{t=0} = 0. \end{cases} \tag{15}$$

A new cost function will be defined for solving the interface problem to the prescribed accuracy and cost effectively. The adjoint problem will be derived similarly.

We use a sensor-grid method [30-32] that is essentially a model reduction method that significantly reduces the computational cost. The basic idea to decrease the cost function as defined over the whole problem domain to a discrete cost function defined over the set of sensor locations. We briefly describe the basic procedure of this method by using the general evolution equation (13).

Assume we have N_s sensor locations $\{X_j\}_{j=1}^{N_s}$ and $\gamma_j(t)$ is the measurement of φ at the location X_j and time t. Given N basis functions $u_i, i = 1, \cdots, N$, assume φ_i is the solution of equation (13) with initial function u_i. Then the sensor-grid method is to find the initial data function

$$u = \sum_{i=1}^{N} \alpha_i u_i$$

to minimize the discrete cost function

$$F(\alpha) = \sum_{j=1}^{N_s}\left(\sum_{i=1}^{N} \alpha_i \varphi_i(X_j, t) - \gamma_j(t)\right)^2 + \sum_{i=1}^{N} \kappa_i (\alpha_i - \beta_i)^2, \tag{16}$$

where $\kappa = (\kappa_1, \cdots, \kappa_N)^T$ contains penalty coefficients for $\beta = (\beta_1, \cdots, \beta_N)^T$, which is an *a priori* vector that is updated during the simulation to achieve the desired accuracy. Define the sensor-grid to be a grid using all the sensor locations as the vertices of the elements in the grid. Then a multi-scale interpolation technique [31-32] is utilized over the sensor-grid for solving the equation during the time evolution iteration.

Applying the sensor-grid method and multiscale interpolation method to the dual porosity Navier-Stokes model entails difficulties that have not been experienced in past Navier-Stokes applications. The constrained optimization components and the interface have a significant impact on the size of the sensor-grid in order to prove stability of the data assimilation process, which requires the analysis for the much more complex system.

V. Conclusions

We have presented a new dual porosity Navier-Stokes, multiphysics model combined with free flow and data assimilation that can be applied to wide variety of real world applications and is more accurate than single porosity models.

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