

## A Redesigned Back Toll Plaza with New Merge Pattern

Yuhao Zhang, Wuyu Zhang, Dan Zhang  
School of Civil Engineering, Qinghai University,  
Xining 810016, China  
e-mail: zhang2680354837@163.com

Fenliang Liu, Shizhe Liu  
School of Civil Engineering, Qinghai University,  
Xining 810016, China  
e-mail: zhang2680354837@163.com

**Abstract**—As the main way of road transportation, highway plays an important role in the development of national economy. The optimum design mathematical model of expressway toll station is established based on the queueing theory. And this paper analyzed the performance of toll plaza, calculated the shape of toll plaza and lane confluence mode. Finally, we use LINGO software to solve the influence of the automatic driving vehicle on the design model.

**Keywords**—Highway; Toll plaza; Queuing Theory; LINGO

### I. INTRODUCTION

In recent years, with the growth of the global economy, the demand for highways has also grown rapidly. As an indispensable part of modern transportations, it plays an irreplaceable role in social and economic development [1]. But one problem has attended simultaneously[2], which is the facilities along the highway can't meet the rapid growth of traffic volume[3]. For example[4], the unreasonable structure and shape of toll plaza[5] frequently cause longer waiting time, more congestion and other problems[6].

In this paper, we establish a reasonable mathematical model to analyze and calculate the optimal redesign from many schemes[7], which make toll plazas cheaper, safer and more efficient. Finally, we quantified influences of the automatic driving vehicle on the redesign model by the LINGO software[8].

### II. NEW DESIGN OF TOLL PLAZA

#### New design of toll plaza

The new form of exit from toll station to normal lane is shown in Fig.1

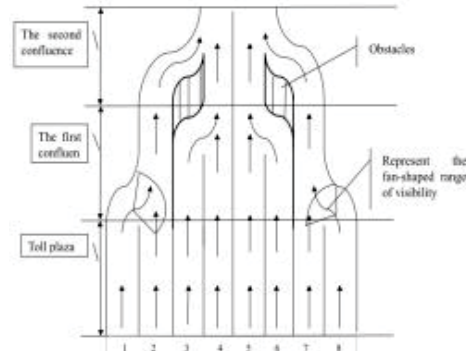


Figure 1. a schematic diagram of the shape and confluence of toll stations

The relevant information has been proved that the easement curve conforms to the driving laws, expands the driver's visual range and appropriately increases the distance between vehicles. Accordingly we replace the hypotenuse of the traditional trapezoidal toll plaza with two reverse-connected detente curves. When passing along easement lanes, the vehicle's merging process becomes safer and more convenient, and traffic capacity of the toll plaza can be enriched during turning time of vehicles.

In redesigned figure 1, Lane confluence is two wider lane that not opposite to the outside roads in the toll plaza (1, 2, 7, 8 lane) some cars in this part merged. As shown in this figure, unlike traditional lanes merge form, we fully consider the prevention of traffic accidents, the adjacent to two lanes merge first, then combined two lanes merge way, to decrease at the beginning of the vehicle into. In order to make the merge area meaningful, the traffic flow should be saturated without congestion, that is, the throughput is less than the road capacity of the combined area. However, in order to reduce the cost of land and road construction, the combined area should be as small as possible [2], so these comprehensive considerations should be taken.

Calculation model: M/M/B queueing model

The number of toll station is defined as, traffic as poisson flow, average arrival rate is defined as, every toll station service rate is defined as (assuming for the same kind of toll station type), after the vehicle number is defined as n, the average service rate for the entire services (when n=B or higher), or (when n < B), as, referred to as a system service strength, system steady state probability:

$$P_n = \begin{cases} \frac{1}{n!} (c\lambda)^n P_0, & n < B \\ \frac{c^B}{c!} \rho^n P_0, & n \geq B \end{cases}$$

$$P_0 = \left[ 1 + \sum_{n=1}^{B-1} \frac{(B\rho)^n}{n!} + \sum_{n=B}^{\infty} \frac{(B\rho)^n}{B!} \frac{1}{B^{n-B}} \right]^{-1} \quad (1)$$

We can get the probability that the vehicle must wait at the toll station:

$$P_w = \sum_{n=B}^{\infty} P_n = \frac{P_0 (B\rho)^B}{(1-\rho)B!} \quad (2)$$

Average waiting time for vehicles:

$$W = \frac{N_w}{B\lambda} = \frac{\rho}{B\lambda(1-\rho)} P_w$$

Average delay of vehicle:

$$T = \frac{1}{\mu} + W = \frac{1}{\mu} + \frac{P_w}{B\mu - B\lambda}$$

Average number of vehicles in the system:

$$N = \frac{B\lambda}{\mu} + N_w = B\rho + P_w \frac{\rho}{1-\rho} \quad (3)$$

The analysis of new design of toll plaza

Easement curve

The gradient curve where we set up is the mitigation curve. The easement curve is a curve that is connected to different linear lines, and the curvature is constantly changing. According to the relevant design specification, it connects the straight line, circle curve and other linear forms to achieve the optimal working state. It is also widely used in highway design. It needs to be compatible with terrain, natural scenery, and provide good driving conditions for drivers in psychological, physiological, and vision, which can ensure the safety of the vehicle traveling [3].

For the design of the easement curve

According to the study of accident rate in foreign, is connected with the following factors: the length of curve is, the radius of the curve is, the easement curve which is represented by. If there is easement curve, . If there is not, . The regression model which people like Zegger developed suggests that [4]:

$$P_s = \frac{1.55L_c + \frac{80.2}{R} - 0.0125}{1.55L_c}$$

As is seen from the formula, the accident rate is negatively related to whether easement curve existed. Therefore, in the reform of the structure design of toll plaza, the traditional trapezoidal vehicle confluence square is replaced by the easement curve in order to guide the vehicle confluence to effectively improve its safety and reduce the accident rate.

The relationship between the length of the easement curve and the visual condition of the driver

According to relevant literature,

$$R \times L = C \quad C = \frac{R^2}{9} \sim R^2 \quad (4)$$

Where:

R describes curve radius,

L describes the length of easement curve,

C describes constant.

The length of the easement curve is [5]:

$$L = \frac{R}{9} \sim R$$

To sum up, the improvement we have done for the toll plaza is reasonable.

$$V = N\beta\gamma Li$$

In terms of road capacity, saturation, number of lanes, length of road sections and length of road sections,

our new design scheme adopts the first two lanes to merge the adjacent lanes, and then merge the new combined lanes by two and two to simplify the former multi-lane merge way into a two-lane merge way. In fact, it can increase the capacity indirectly by increasing the lane reduction coefficient.

TABLE I. THE REDUCTION COEFFICIENT OF VEHICLES

lane	one	two	three	four
Reduction coefficient	1	0.8-0.89	0.65-0.79	0.50-0.65

The area of the designed part

We can determine that the area of the toll plaza from the tollbooth to the normal lane is , and it's a direct line to the cost of construction. It directly determines the cost of construction. According to the regulation, the length of the transition segment 1 can be obtained after the gradual changed rate of the easement curve of toll plaza can be determined. The width of toll plaza is determined by the product of the number of toll booths and the width of the driveway.

According to the optimization of queuing system: in the queuing system, the more vehicles pass, the higher the service efficiency, the shorter the stay time is, the loss is least. So the toll station will increase the number of staff and equipment, but the toll station cannot be invested indefinitely. Therefore, it is necessary to optimize the design, the purpose of which is to minimize the cost of traffic while maintaining a certain traffic capacity.

Under the assumption of steady state, the total operating cost is, and the optimal value of B is, and then is the minimum cost. Since B can only take integer, that is, is a discrete function, so it can only be solved by the method of marginal analysis. In fact, according to the minimum value of , there can be [6]:

$$\begin{cases} f(B_0) \leq f(B_0 - 1) \\ f(B_0) \leq f(B_0 + 1) \end{cases}$$

Simplification and consolidation:

$$L_2(B_0) - L_2(B_0 + 1) \leq \frac{e_1}{e_2} \leq L_2(B_0 - 1) - L_2(B_0)$$

Through the LINGO software simulation. The difference between the two adjacent items is to see which constant between the two, so as to determine the optimal solution for the traffic cost and capacity to reach the optimal number of tolling booths B.

The shape and size of the merge area affects the way the vehicles change, in the meantime, to reduce the costs of land and construction roads, the area of it should be as small as possible in the design of the merge area. Taking the new scheme we designed as an example, assuming that , the linear distance between the first and the eighth lane toll lanes is 1, then the linear distance between the second and the seventh lane is 2l, and the distance of the other toll lanes from the toll booth to the merge end is 3l, the width of each line lane is delta w, and the average width of the easement curve lane is , Area of the new scheme design part:

$$S = S_{exit} + S_{confluence} - 2S_{obstacle}$$

Using the same method to analyze the existing traditional toll plaza:

$$\begin{aligned} S &= S'_{exit} + S'_{confluence} = 16l\Delta W + 4l\Delta W_{confluence} \\ S'_{exit} &= (8 \times \Delta W_{1-2}) \times l \\ S'_{confluence} &= (4 \times \Delta W_{1-2} + 2 \times \Delta W_{confluence}) \times 2l \end{aligned}$$

Thus, the improved scheme requires small road area and low cost.

### III. THE IMPACT OF THE INCREASE IN SELF-DRIVING VEHICLES

#### Establishment of model

Jian Minxing et al. research conclusion [7] showed that, under the same traffic density, the safe distance of self-driving car request is shorter and almost needn't the reaction time, also won't random moderated. What's more, self-driving cars will change lane as long as meet the conditions for a lane change along with the increase of the proportion of self-driving vehicles gradually increased the average speed of the whole section, the traffic flow will also increase gradually, which will greatly improve the entire sections of the average speed. So under the same density of traffic, traffic flow will increase when the average speed of traffic increased.

The change of the proportion of self-driving vehicles leads to the change of road traffic flow, and the average arrival rate of the vehicle at the toll station varies with the proportion of the auto-pilot. The average arrival rate of the vehicle at toll station is:

$$\lambda = \alpha \cdot \lambda_1 + (1 - \alpha) \times \lambda_2 \quad (5)$$

According to the actual situation, the proportion of self-driving vehicles often are converted into the number of charging station for self-driving vehicles the proportion of the corresponding automatic car driving for 1/B, 2/B, ..., 1.

We determine the number of the ETC toll booths by the proportion of self-driving vehicles and consider the operation cost to meet the ideal result. There is the formula calculating the operation cost [8]:

$$F = B e_1 \mu + e_1 \left[ B \rho + \frac{P_0(B\rho)^B}{(1-\rho)B!} \times \frac{\rho}{1-\rho} \right] \quad (6)$$

#### Solution of model

Following illustrations is the model' effect:

The average arrival rate of vehicles is  $\lambda=60$ , the cost of the loss of a vehicle's stay is  $e1=6$ . The service (inspection) time obeys the negative exponential distribution, and average service rate is  $\mu=100$ , the service cost of each set of toll booths is  $e2=0.3$ .

In this case, self-driving vehicles' average arrival rate is  $\lambda=80$  and the ETC toll booth's average service

rate is  $\mu=400$ , so the service cost of each toll booth is  $e2\mu=35$ . The traffic capacity has been greatly improved.

According to the numerical proportion of the above model, the calculation results can be obtained by the LINGO algorithm program operation. The corresponding calculation results can be obtained from the proportion of different self-driving vehicles:

TABLE II. CORRESPONDING DATA FOR DIFFERENT PROPORTIONS OF SELF-DRIVING VEHICLES

Proportion of self-driving vehicles	1/B	2/B	3/B	4/B	...	1
Average arrival rate	60+20/B	60+40/B	60+60/B	60+80/B	...	80
Optimal number of the toll booth	2	2	2	2	...	2
Minimum cost	64.8	65.7	66.8	68.0	...	65.7

The above data in the case shows that the optimal number of toll booths is basically the same, all of which are 2, but the minimum cost is changed correspondingly with its tendency.

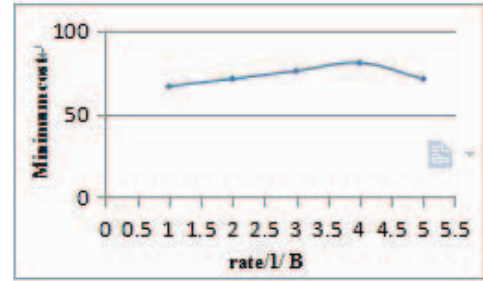


Figure 2. Relationship between proportion of self-driving vehicles and change trend of minimum cost

As shown in the fig 2, the minimum cost has a linear trend with the change of the proportion of the self-driving vehicles. The data above shows that 2 toll booths are optimal, and the cost will be minimal when self-driving vehicles' proportion is 100%.

Accordingly, we can change our design according to the actual situation of self-driving vehicles', and apply it to practice better.

### IV. CONCLUSIONS

The capability of toll booths are interrelated with each other and its influencing factors are complex. We seize the major factors to analysis. Today, toll stations are numerous and growing. At design time we consider that proposal

shouldn't affect the ability to accident prevention and the traffic capacity, but the real costs in construction and operating should reduce as far as possible. So we redesign a type of toll station with new shape, size and merge pattern to reduce the construction cost. Also we conducted data analysis on the new toll plaza, and the results show that the new type of toll plaza has high safety performance and reasonable design, and the improved scheme requires small road area and low cost..

#### ACKNOWLEDGMENT

This study was supported by S&T Foundation Platform of Qinghai Province (Grants No. 2018-ZJ-T01), The Cooperation Program of Qinghai Province (Grants No.2017-HZ-804)

#### REFERENCES

- [1] Huang Liyan. Discussing the Importance of Highway Construction on Economic Development [J]. Foreign In Vestment In China,2012(24):243+245
- [2] Lu Yang, Zhang Xueya, Yang Zongyue. Study on the improvement of highway lane merging[J]. China High-tech,2017,1(02):32-34
- [3] Xu Lucheng. The Design and Application of Road's Transition Curves[J]. Sci-Tech Information Development & Economy, 2006(02): 279-280
- [4] Li Changcheng, Kan Weisheng. Discussion on Relationship between Expressway Crash and Horizontal Alignment [J]. Journal of Highway and Transportation Research and Development, 2007(01): 126-126+146
- [5] Lai Kunping. Discussing the design of highway detente curve [J]. Communications Science and Technology Heilongjiang, 2015,38(07):44-45
- [6] Han Zhonggeng. Mathematical Modeling Method and Application (second edition) [M].Higher Education Press for Organizing The Contest. 2013:271-272
- [7] Xing Jianmin, Li Haonan, Duan Deyu. Automated Driving Car in Traffic in the Network Performance Analysis[J].Mathematical Modeling And Its Applications,2017,(6)2
- [8] Jiang Shuhua, Fu Xiaoliang. Application of Checking Service Model of Supermarket Based on Queuing Theory [J]. Logistics Sci-Tech No.10, 2008(10):141-142.