Simultaneous state and fault estimation of discrete-time Markovian jump stochastic systems: The event-triggered design

Xu Liu, Yunji Li
Engineering Research Center of Internet of Things
Technology Applications Ministry of Education
Jiangnan University
Wuxi, People's Republic of China
liyunji2015@163.com

Li Peng
Jiangsu Key Laboratory of IOT Application Technology
Taihu University of Wuxi
Wuxi, People's Republic of China
jnpengli@126.com

***Abstract*—In this paper, the state and fault estimation problem is investigated for a class of discrete-time Markovian jump systems with unknown disturbances and actuator faults. The unknown disturbances, satisfying the zero-mean Gaussian distribution, are introduced to reflect the limited capacity of the communication networks resulting from the noisy environment. On the one hand, a novel event-triggered fault estimator and the corresponding event condition are proposed where the task is to reconstruct both system states and actuator faults under the aperiodic data transmission. On the other hand, estimator gains are derived using the stochastic stability in terms of a set of standard linear matrix inequalities. Finally, a simulation example is given to illustrate the usefulness of the developed event-triggered fault estimation approach.**

***Keywords*-Simultaneous state and fault estimation, Markovian jump systems, Event-triggered data transmission**

I. Introduction

Issues dealing with the health of complex dynamical systems have received much attention and fruitful literature has reported on both the theoretical research and practical applications. It is not difficult to find that majority of the research has dealt with the model-based fault detection, estimation and accommodation problem. In general, most real systems are subjected to the random phenomenon and require a stochastic model. Markovian jump systems are always described by a set of stochastic dynamic processes, containing a series of linear or nonlinear systems with the transition probabilities between the modes, which are defined by a Markov chain with values in a finite set. In recent years, the study of reliability for Markovian jump systems has received a growing attention. For example, Literature [1] studied the problem of robust fault-detection filter for a class of nonlinear stochastic time-delayed Markov jump systems using Takagi–Sugeno fuzzy models. By means of the energy norm indices, the fault detection problem was investigated in [2] for discrete-time Markovian jump systems subject to randomly varying nonlinearities and sensor saturation, where the transition probability matrix was required to have partially unknown entries, whereas the completely known and completely unknown transition probabilities were also investigated as two special cases. A similar study about fault estimation for time-varying Markovian jump systems was presented and analyzed in [3] with the assumption that the considered system is constrained by time-varying distributed delays.

On another research front, in parallel with the rapid development of wireless transmission technology, remote estimation and control problems are frequently encountered in many situations [4]. Unlike the traditional wired estimation and control, the wireless sensors are typically powered by small batteries which are difficult to replace. Overcoming this constraint may lead to the development of an event-triggered data-transmission scheme via reducing number of data-transmission actions so as to achieve a desired tradeoff between energy efficiency and system performance. Event-triggered control and estimation have been recently proposed (see [5] and the reference therein). Similar to control and estimation problems, the event-triggered transmission scheme has also been applied to remote fault diagnosis technique. In [6], the synthesized design of reduce-order fault-detection filter and fault estimator were presented for stochastic systems under an event-triggered sensor transmission scheme, where an upper bound of error covariance is minimized by properly choosing the estimator gains at each time instant. A variance-constrained fault estimator was designed and analyzed in [7] over multi-hop relay networks subject to incomplete information. However, compared to a great deal of research progress for event-triggered fault diagnosis, the problem of event-triggered fault estimation and accommodation for Markovian jump systems still exists an amount of space and needs to be thoroughly probed. Therefore, there is a strong incentive for us to investigate fault estimation and accommodation for Markovian jump systems. In view of these, we will address the problem of simultaneous state and fault estimation for discrete-time Markovian jump systems subject to external stochastic disturbances satisfying the Gaussian distribution. In this work, the main contributions that are worth emphasizing are summarized in the following three aspects.

(1) A unified framework is established within which the event-triggered fault estimation problem can be conveniently handled for discrete-time Markovian jump stochastic systems.

(2) The proposed event-triggered fault estimator can obtain robust reconstruction of system states and actuator

978-1-5386-7445-1/18/$31.00 ©2018 IEEE
DOI 10.1109/DCABES.2018.00089

faults, simultaneously. A sufficient condition is established on the stochastic stability by stochastic Lyapunov function and the estimator gains can be calculated in terms of linear matrix inequalities (LMIs) techniques.

II. Problem statements

In this paper, we consider the following class of discrete-time Markovian jump stochastic systems:

$$\begin{cases} x_{k+1} = A(r_k)x_k + B(r_k)(u_k + f_k) + D_1(r_k)w_k \\ y_k = C(r_k)x_k + D_2(r_k)w_k \end{cases} \tag{1}$$

In above equations, the state $x_k \in \mathbb{R}^{nx}$, the control input $u_k \in \mathbb{R}^{nu}$, the measurement $y_k \in \mathbb{R}^{ny}$, the noise signal $w_k \in \mathbb{R}$ which is assumed to be a zero-mean Gaussian white noise sequence with $\mathbb{E}\left[w_k^2\right] \leq b_1$. The actuator fault $f_k \in \mathbb{R}^{nf}$ satisfying the following norm bounded constraint:

$$\|f_k\| \leq b_2 \tag{2}$$

The variable r_k is a limited model Markov chain in model space $S = \{1,\ 2,\ \dots,\ s\}$ with transition probability matrix $\psi = [\lambda_{ij}]$, described by

$$\mathrm{Prob}\{r_{k+1} = j \,|r_k = i\} = \lambda_{ij},\ \forall i, j \in S \tag{3}$$

where $\lambda_{ij} \geq 0\ (i, j \in S)$ is the mode transition probability from i to j and $\sum_{j=1}^{s} \lambda_{ij} = 1$, $\forall i \in S$. For presentation convenience, we denote the following constant matrices with appropriate dimensions for each possible value of $r_k = i$:

$$\begin{aligned} &A(r_k) = A_i,\ B(r_k) = B_i,\ D_1(r_k) = D_{1,i}, \\ &C(r_k) = C_i \text{ and } D_2(r_k) = D_{2,i} \end{aligned} \tag{4}$$

Before proceeding further, it is necessary to introduce the notion of stochastic stability.

Definition 1. *[Definition 2, [8]] A discrete stochastic process ξ_k is said to be stochastically stable, if for any initial state $(\xi_0,\ r_0)$, the following condition holds*

$$\mathbb{E}\left[\sum_{k=0}^{\infty} \|\xi_k\|^2 \mid \xi_0,\ r_0\right] < \infty \tag{5}$$

III. Design of fault estimator based on event-triggered scheme

As discussed in Section I, the measurements are not sent to the remote estimator at every time step in an event-triggered implementation for saving energy. Hence, the event condition can be defined by using the output vectors

$$g_k = (y_{k,l} - y_k)^T (y_{k,l} - y_k) \leq \delta_1^2 \tag{6}$$

where y_k is the measurement at time k and $y_{k,l}$ is the broadcast measurement at latest event time. The threshold of the event-triggered condition $\delta_1 > 0$. The events are triggered as long as the condition is satisfied. The event triggering instants $0 \leq \ell_0 \leq \ell_1 \leq \ldots \leq \ell_s \leq \ldots$ can be denoted as

$$\ell_{s+1} = \min\{k \in \mathbb{N} \,|k > \ell_s,\ g_k > 0\} \tag{7}$$

In this article, we are interested in designing the following event-triggered fault estimator with the time period $k \in [\ell_s,\ \ell_{s+1}]$

$$\begin{cases} \hat{x}_{k+1} = A_i\hat{x}_k + K(r_k)(y_{k,l} - C_i\hat{x}_k) + B_i\left(u_k + \hat{f}_k\right) \\ \hat{f}_{k+1} = \hat{f}_k + L(r_k)(y_{k,l} - C_i\hat{x}_k) \end{cases} \tag{8}$$

where $\hat{x}_k$ and $\hat{f}_k$ are the estimated system state and the estimated actuator fault, respectively. The matrices $K(r_k)$ and $L(r_k)$ are estimator gains to be determined, which are also simplified as K_i and L_i, respectively. The fault and state estimation errors dynamics can be obtained by (1) and (8)

$$\begin{aligned} e_{k+1} &= x_{k+1} - \hat{x}_{k+1} \\ &= (A_i - K_iC_i)e_k + B_ie_{f,k} + D_{1,i}w_k \\ &- K_iD_{2,i}w_k - K_i\Delta_k \end{aligned} \tag{9}$$

$$\begin{aligned} e_{f,k+1} &= f_{k+1} - \hat{f}_{k+1} \\ &= f_{k+1} - \hat{f}_k - L_i(y_k + \Delta_k - C_i\hat{x}_k) \\ &= \Delta_{f,k} + e_{f,k} - L_i(C_ix_k + D_{2,i}w_k - C_i\hat{x}_k + \Delta_k) \\ &= \Delta_{f,k} + e_{f,k} - L_iC_ie_k - L_iD_{2,i}w_k - L_i\Delta_k \end{aligned} \tag{10}$$

where $e_{k+1} = x_{k+1} - \hat{x}_{k+1}$, $e_{f,k} = f_k - \hat{f}_k$, $\Delta_k = y_{k,l} - y_k$, and $\Delta_{f,k} = f_{k+1} - f_k$.

Now we are in a position to analyze the stochastic stability on fault and state estimation errors dynamics and design the corresponding estimator gains in terms of a set of LMIs.

Theorem 1. *Consider the discrete-time Markovian jump system (1) with the given event condition (6). Let the positive scalars b_1, b_2 and δ_1 be given. The dynamics (9) and (10) are stochastically stable, assuming that there a set of positive definite symmetric matrices P_i^1, P_i^2 and arbitrary matrices $K_i = \left(\bar{P}_i^1\right)^{-1}R_i^1$, $L_i = \left(\bar{P}_i^2\right)^{-1}R_i^2$ $(i \in S)$ satisfying*

$$\Lambda_i = \begin{bmatrix} \Lambda_{1,i} & \Lambda_{2,i} \\ * & \Lambda_{3,i} \end{bmatrix} < 0 \tag{11}$$

where

$$\Lambda_{1,i} = diag\left[-P_i^1,\ -P_i^2,\ -\frac{1}{b_1}I,\ -\frac{1}{\delta_1}I,\ -\frac{1}{b_2}I\right] \tag{12}$$

$$\Lambda_{2,i} = \begin{bmatrix} \left(\bar{P}_i^1A_i - R_i^1C_i\right)^T & -C_i^T\left(R_i^2\right)^T \\ B_i^T\left(\bar{P}_i^1\right)^T & \left(\bar{P}_i^2\right)^T \\ \left(\bar{P}_i^1D_{1,i} - R_i^1D_{2,i}\right)^T & -D_{2,i}^T\left(R_i^2\right)^T \\ -\left(R_i^1\right)^{-1} & -\left(R_i^2\right)^T \\ 0 & \left(\bar{P}_i^2\right)^T \end{bmatrix} \tag{13}$$

$\Lambda_{3,i} = diag\{-\bar{P}_i^1,\ -\bar{P}_i^2\}$, $\bar{P}_i^1 = \sum_{j=1}^{s}\lambda_{ij}P_j^1$ and $\bar{P}_i^2 = \sum_{j=1}^{s}\lambda_{ij}P_j^2$. Moreover, the estimator gains can be calculated by $K_i = \left(\bar{P}_i^1\right)^{-1}R_i^1$ and $L_i = \left(\bar{P}_i^2\right)^{-1}R_i^2$, $\forall i \in S$.

Proof: Define the following stochastic Lyapunov function:

$$V(r_k) = V_1(r_k) + V_2(r_k) \tag{14}$$

where $V_1(r_k) = e_k^T P^1(r_k) e_k = e_k^T P_i^1 e_k$ and $V_2(r_k) = e_{f,k}^T P^1(r_k) e_{f,k} = e_{f,k}^T P_i^2 e_{f,k}$. Then along the trajectories of (9) and (10), we have

$$\begin{aligned}
&\mathbb{E}\{\Delta V_1(r_k)\} = e_k^T (A_i - K_i C_i)^T \bar{P}_i^1 (A_i - K_i C_i) e_k \\
&+ e_{f,k}^T B_i^T \bar{P}_i^1 B_i e_{f,k} \\
&+ w_k^T (D_{1,i} - K_i D_{2,i})^T \bar{P}_i^1 (D_{1,i} - K_i D_{2,i}) w_k \\
&+ \Delta_k^T K_i^T \bar{P}_i^1 K_i \Delta_k + 2 e_k^T (A_i - K_i C_i) \bar{P}_i^1 B_i e_{f,k} \\
&- 2 e_k^T (A_i - K_i C_i) \bar{P}_i^1 K_i \Delta_k - e_{f,k}^T B_i^T \bar{P}_i^1 K_i \Delta_k \\
&- e_k^T P_i^1 e_k
\end{aligned} \tag{15}$$

and

$$\begin{aligned}
&\mathbb{E}\{\Delta V_2(r_k)\} = \Delta_{f.k}^T \bar{P}_i^2 \Delta_{f,k} + e_{f,k}^T \bar{P}_i^2 e_{f,k} \\
&+ e_k^T C_i^T L_i^T \bar{P}_i^2 L_i C_i e_k + w_k^T D_{2,i}^T L_i^T \bar{P}_i^2 L_i D_{2,i} w_k \\
&+ \Delta_k^T L_i^T \bar{P}_i^2 L_i \Delta_k + 2\Delta_{f,k}^T \bar{P}_i^2 e_{f,k} - 2\Delta_{f,k}^T \bar{P}_i^2 L_i C_i e_k \\
&- 2\Delta_{f,k}^T \bar{P}_i^2 L_i \Delta_k - 2 e_{f,k}^T \bar{P}_i^2 L_i C_i e_k \\
&- 2 e_{f,k}^T \bar{P}_i^2 L_i \Delta_k + 2 e_k^T C_i^T L_i^T \bar{P}_i^2 L_i \Delta_k - e_{f,k}^T P_i^2 e_{f,k}
\end{aligned} \tag{16}$$

where $\bar{P}_i^1 = \sum_{j=1}^{s} \lambda_{ij} P_j^1$ and $\bar{P}_i^2 = \sum_{j=1}^{s} \lambda_{ij} P_j^2$. By noting that assumption (2), event condition (6) and $\mathbb{E}[w_k^2] \le b_1$, we have

$$\begin{aligned}
&-\frac{1}{\delta_1}\Delta_k^T \Delta_k + \delta_1 > 0,\ -\frac{1}{b_1} w_k^T w_k + b_1 > 0 \\
&and\ -\frac{1}{b_2}\Delta_{f,k}^T \Delta_{f,k} + b_2 > 0
\end{aligned} \tag{17}$$

According to (17), it can be obtained further that

$$\begin{aligned}
\mathbb{E}\{\Delta V(r_k)\} &\le \mathbb{E}\{\Delta V_1(r_k)\} + \mathbb{E}\{\Delta V_2(r_k)\} \\
&- \frac{1}{\delta_1}\Delta_k^T \Delta_k + \delta_1 - \frac{1}{b_1} w_k^T w_k \\
&+ b_1 - \frac{1}{b_2}\Delta_{f,k}^T \Delta_{f,k} + b_2
\end{aligned} \tag{18}$$

By considering the Definition 1, it is easily known that

$$\begin{aligned}
\mathbb{E}\{\Delta_k\} &< -\lambda_{\min}(-\Lambda_i)\|\rho_k\|^2 \\
&+ \tilde{b} < -\lambda_{\min}(-\Lambda_i)\|\tilde{e}_k\|^2 + \tilde{b} < \infty
\end{aligned} \tag{19}$$

where $\tilde{b} = b_1 + b_2 + \delta_1$, $\rho_k = \begin{bmatrix} e_k^T & e_{f,k}^T & w_k^T & \Delta_k^T & \Delta_{f,k}^T \end{bmatrix}^T$ and $\tilde{e}_k = \begin{bmatrix} e_k^T & e_{f,k}^T \end{bmatrix}^T$. Obviously, we can get that $\mathbb{E}\{\Delta_k\} < 0$, if $\tilde{b} < \lambda_{\min}(-\Lambda_i)\|\tilde{e}_k\|^2$. The trajectory of $\tilde{e}_k$ will converge to the small set $\Psi = \left\{\tilde{e}_k \,\middle|\, \|\tilde{e}_k\|^2 \le \frac{\tilde{b}}{\lambda_{\min}(-\Lambda_i)}\right\}$. Therefore, $\tilde{e}_k$ is ultimately bounded. This completes the proof. ■

Remark 1. *It is necessary to point out that a general event-triggered fault estimation framework is established for reflecting the engineering practice and examining the influence from event-triggered transmission scheme onto the estimator performance. The available information about the boundedness of stochastic noises b_1, fault b_2, and event condition δ_1 has been reflected to further enhance the flexibility and solvability of the proposed estimator algorithm.*

IV. Numerical Example

In order to test the effectiveness of the proposed event-triggered fault estimation, we provide a simulation example, which is borrowed from [5]. Consider the following transition probability matrix of the Markov process:

$$\psi = \begin{bmatrix} 0.3 & 0.7 \\ 0.4 & 0.6 \end{bmatrix}$$

Suppose that the system involves two modes and the other system parameters are given as follows:

$$A_1 = \begin{bmatrix} -0.6 & 0.4 \\ 0.3 & 0.5 \end{bmatrix},\ A_2 = \begin{bmatrix} 0.3 & 0.5 \\ 0.4 & 0.5 \end{bmatrix},$$

$$B_1 = \begin{bmatrix} 0.5 \\ 0.7 \end{bmatrix},\ B_2 = \begin{bmatrix} 0.45 \\ 0.6 \end{bmatrix},$$

$$C_1 = \begin{bmatrix} 0 & 0.5 \end{bmatrix},\ C_2 = \begin{bmatrix} 0.2 & 0.2 \end{bmatrix},$$

$$D_{11} = \begin{bmatrix} -0.1 \\ 0.7 \end{bmatrix},\ D_{12} = \begin{bmatrix} 0.1 \\ -0.5 \end{bmatrix},$$

$$D_{21} = D_{22} = 0.4$$

The parameters b_1, b_2, δ_1 and δ_2 are taken as 0.2, 1, 0.1, and 0.15, respectively. A time-varying fault is created as

$$f_k = \begin{cases} 0.3 & if\ k \le 20 \\ 0.7k + 0.2\sin k & otherwise \end{cases} \tag{20}$$

With the aforementioned parameters, the inequality in (11) is solved by using the MATLAB, the desired estimator and controller gains can be obtained as follows:

$$K_1 = \begin{bmatrix} 0.71 \\ 0.22 \end{bmatrix},\ K_2 = \begin{bmatrix} 0.0051 \\ -0.1966 \end{bmatrix},$$

$$L_1 = 0.38,\ L_2 = 0.018$$

$$\bar{K}_1 = \begin{bmatrix} 0.31 & 0.478 \end{bmatrix},\ \bar{K}_2 = \begin{bmatrix} 0.12 & 0.97 \end{bmatrix}$$

In the simulation, the stochastic disturbance w_k obeys the zero-mean Gaussian distribution with the variance 0.2. The simulation results are shown in Figures 1-2, where the jump modes are shown in Figure 1. To compare the fault estimation performance, the fault estimation error $\|e_{f,k}\|$ is used. In Figure 2, the proposed event-triggered fault estimator (ETFE) is compared with the traditional time-driven fault estimator (TDFE). From the simulation results in Figures 1-2, it can be observed that the event-triggered data transmission scheme could reduce the communication load without degrading the system performance severely.

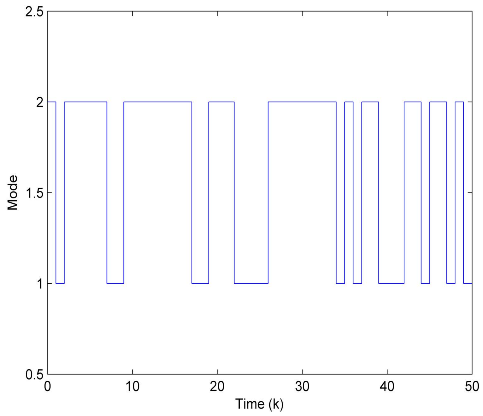

Figure 1. Markov jump modes.

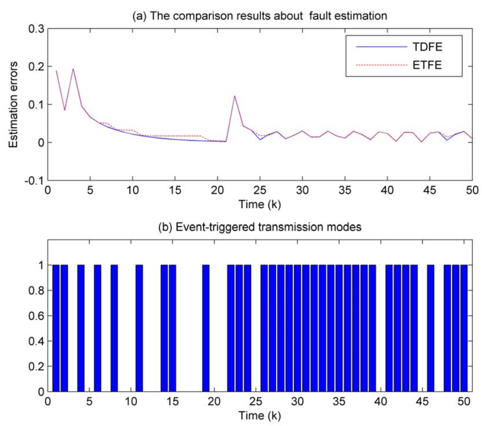

Figure 2. The proposed event-triggered fault estimator (ETFE) is compared with the traditional time-driven fault estimator (TDFE).

V. Conclusion

This paper proposed the synthesized design of fault estimator and event-triggered data transmission scheme for a class of discrete-time Markovian jump systems with unknown disturbances and actuator faults. The unknown disturbances were assumed to be random according to a stochastic variable satisfying the Gaussian distribution. Estimator gains are derived using the stochastic stability in terms of a set of standard linear matrix inequalities. Simulation results showed that the event-triggered data transmission scheme could reduce the communication load without degrading the system performance severely.

Acknowledgment

This work is supported by Education Ministry and China Mobile Science Research Foundation: Video Big Data Analysis and IoT Interaction Technology Research (MCM20170204).

References

[1] S. P. He and F. Liu, Fuzzy model-based fault detection for Markov jump systems, *Int. J. Robust Nonlinear Control* , vol. 19, pp. 1248-1266, 2009.

[2] H. Dong, Z. Wang, and H. Gao, Fault detection for markovian jump systems with sensor saturations and randomly varying nonlinearities, *IEEE Trans. Circuits Syst. I Regul. Pap.* , vol. 59, no. 10, pp. 2354-2362, 2012.

[3] W. Ren, C. Wang, and Y. Lu, Fault estimation for time-varying Markovian jump systems with randomly occurring nonlinearities and time delays, *J. Franklin Inst.* , vol. 354, no. 3, pp. 1388-1402, 2017.

[4] Y. Li and L. Peng, Event-triggered sensor data transmission policy for receding horizon recursive state estimation, *J. Algorithm. Comput. Technol.* , vol. 11, no. 2, pp. 178-185, 2017.

[5] Y. Li, P. Li, and W. Chen, An energy-efficient data transmission scheme for remote state estimation and applications to a water-tank system, *ISA Trans.* , vol. 70, pp. 494-501, 2017.

[6] Y. Li, Q. Wu, and L. Peng, Simultaneous Event-Triggered Fault Detection and Estimation for Stochastic Systems Subject to Deception Attacks, *Sensors* , vol. 18, no. 2, p. 321, 2018.

[7] Y. Li and L. Peng, Event-Triggered Fault Estimation for Stochastic Systems over Multi-Hop Relay Networks with Randomly Occurring Sensor Nonlinearities and Packet Dropouts, *Sensors* , vol. 18, no. 3, p. 731, 2018.

[8] Z. G. Wu, P. Shi, H. Su, and J. Chu, Asynchronous $l_2 - l_\infty$ filtering for discrete-time stochastic Markov jump systems with randomly occurred sensor nonlinearities, *Automatica* , vol. 50, no. 1, pp. 180-186, 2014.